Verification of SHSLBM for flow around a square cylinder

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Lattice Boltzmann Method

- The LBM for incompressible fluid flows with BGK approximation

$$f_\alpha(\boldsymbol{r}+\boldsymbol{e}_\alpha\,\delta_t, t+\delta_t) = f_\alpha(\boldsymbol{r},t) + \frac{f_\alpha^{eq}(\boldsymbol{r},t) - f_\alpha(\boldsymbol{r},t)}{\tau_v}, \alpha = 0,1,\ldots N-1$$

$$\rho = \sum_{\alpha=0}^{N-1} f_\alpha, \qquad \rho\boldsymbol{u} = \sum_{\alpha=0}^{N-1} f_\alpha \boldsymbol{e}_\alpha$$

$$f_\alpha^{eq}(\boldsymbol{r},t) = \rho\omega_\alpha\left[1 + \frac{\boldsymbol{e}_\alpha.\boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{e}_\alpha.\boldsymbol{u})^2 - (c_s|\boldsymbol{u}|)^2}{2\,c_s^4}\right]$$

$$\mu = \left(\tau_v - \frac{1}{2}\right)\rho c_s^2 \delta_t$$

The equilibrium distribution functions also satisfy the conservation of mass and momentum at any physical location:

$$\rho = \sum_{\alpha=0}^{N-1} f_\alpha^{eq}, \qquad \rho\boldsymbol{u} = \sum_{\alpha=0}^{N-1} f_\alpha^{eq}\boldsymbol{e}_\alpha$$

For D2Q9:

$$e_\alpha = \begin{cases} 0, & \alpha = 0 \\ \cos\left[\frac{(\alpha-1)\pi}{2}\right]c_s, \sin\left[\frac{(\alpha-1)\pi}{2}\right]c_s, & \alpha = 1,2,3.4 \\ \sqrt{2}\cos\left[\frac{(\alpha-5)\pi}{2}+\frac{\pi}{4}\right]c_s, \sqrt{2}\cos\left[\frac{(\alpha-5)\pi}{2}+\frac{\pi}{4}\right]c_s, & \alpha = 5,6,7,8 \end{cases}$$

Simplified and Highly Stable LBM

- Simplified and Highly Stable LBM(SHSLBM) is a relatively new scheme.
- Use fractional step method to convert the governing equations into predictor and corrector steps.
- Use only equilibrium distribution functions for the evolution of macroscopic variables.

Predictor step:

$$\rho^* = \sum_\alpha f_\alpha^{eq}(\boldsymbol{r}-\boldsymbol{e}_\alpha\delta_t, t-\delta_t)\ ; \quad \rho^*\boldsymbol{u}^* = \sum_\alpha \boldsymbol{e}_\alpha f_\alpha^{eq}(\boldsymbol{r}-\boldsymbol{e}_\alpha\delta_t, t-\delta_t)$$

Corrector step:

$\rho(\boldsymbol{r},t) = \rho^*$, also

$$\rho(\boldsymbol{r},t)\boldsymbol{u}(\boldsymbol{r},t) = \rho^*\boldsymbol{u}^* + (\tau_v - 1)\left(\sum_\alpha \boldsymbol{e}_\alpha f_\alpha^{eq}(\boldsymbol{r}+\boldsymbol{e}_\alpha\delta_t, t)\right) - (\tau_v - 1)\rho(\boldsymbol{r}, t-\delta_t)\boldsymbol{u}(\boldsymbol{r}, t-\delta_t)$$

Advantages of SHSLBM

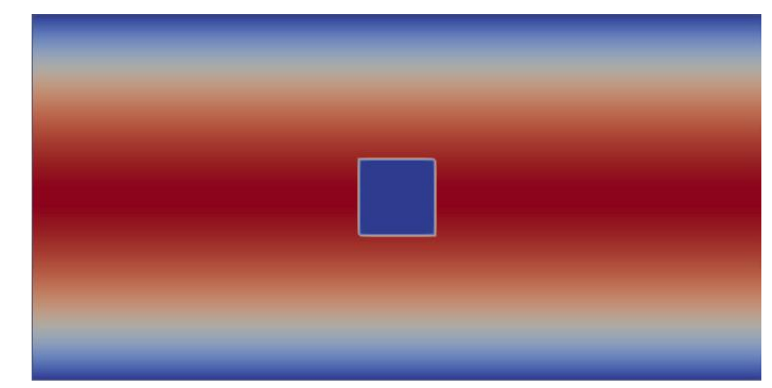
- Physical boundary conditions can be applied like a typical finite volume scheme.
- Non-uniform mesh can also be used, which is not possible using traditional LBM.
- As it uses only equilibrium distribution function, the memory cost is reduced.
- The scheme is highly stable even for a value of relaxation parameter very close to 0.5.

OBJECTIVES

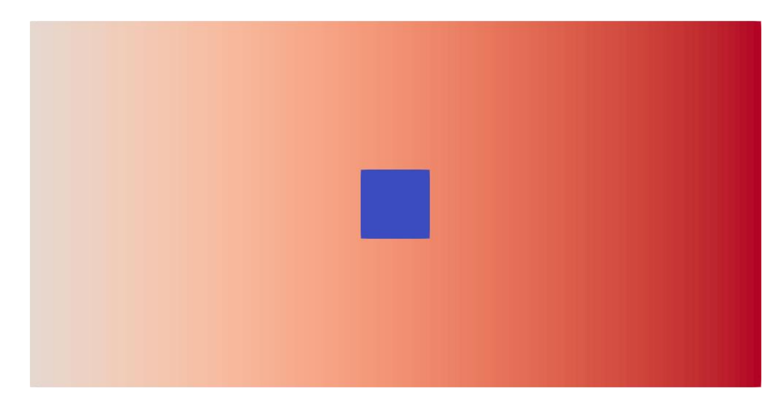
- The objective of this work is to verify the SHSLBM for flow around a square cylinder problem, especially the effect of initial condition on the simulation results.

METHODOLOGY

- The computational domain is initialized in two different ways
  1. The density field is initialized with a constant density.
  2. The density field is initialized with the analytical density obtained from the geometry, without the square cylinder (Poiseuille flow).
- The velocity field for both the cases are initialized with the analytical velocity of the Poiseuille flow.
- The cases considered includes both the steady and unsteady cases.
- The drag coefficient values are used for the comparison.
- No-slip boundary condition is used for the square cylinder.



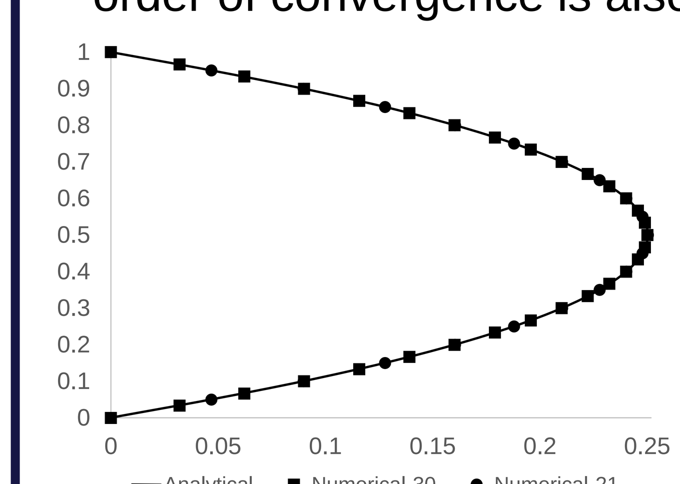
Velocity initialization

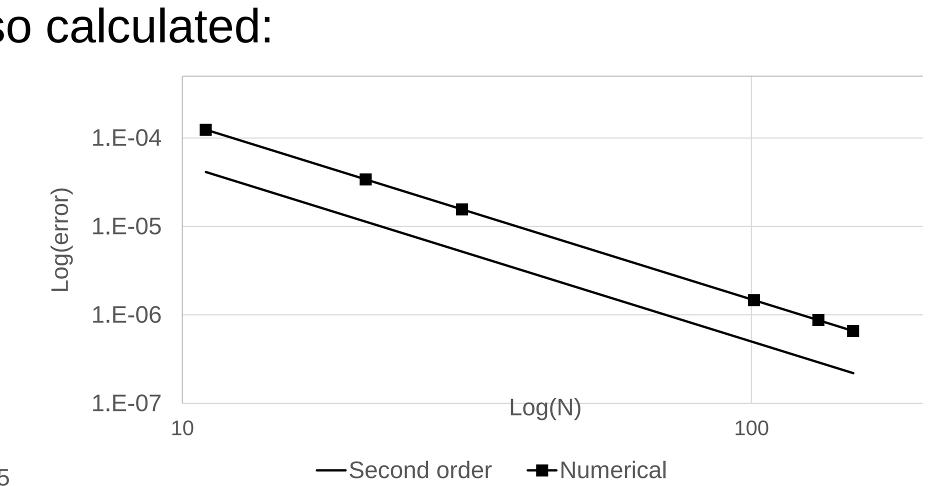


Density initialization

VERIFICATION

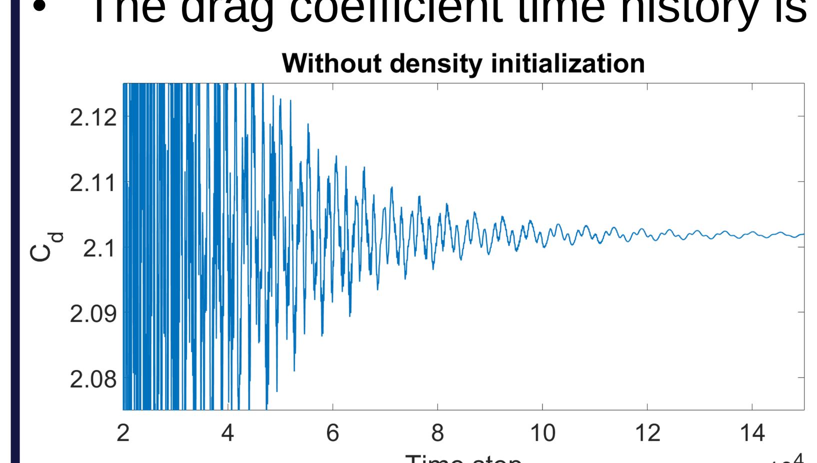
- The solver is validated and verified using a simple case of Poiseuille flow and the results are compared to the analytical solution and the order of convergence is also calculated:

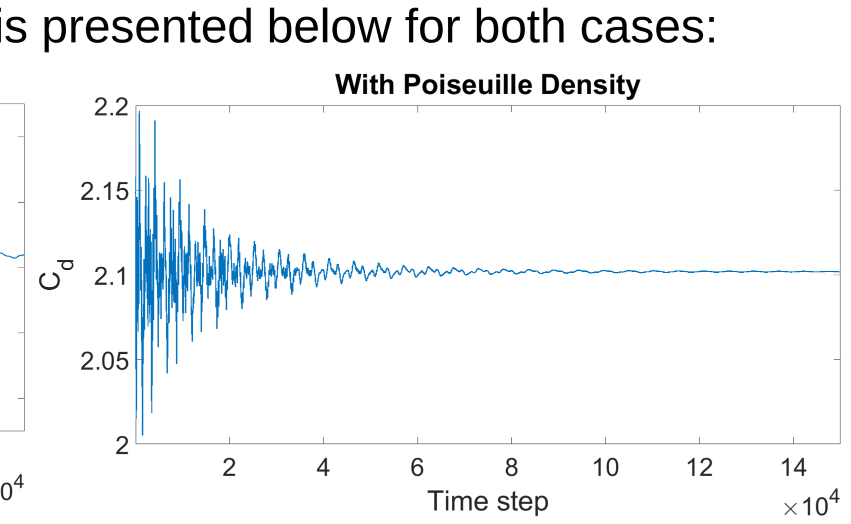




RESULTS

- The drag coefficient time history is presented below for both cases:





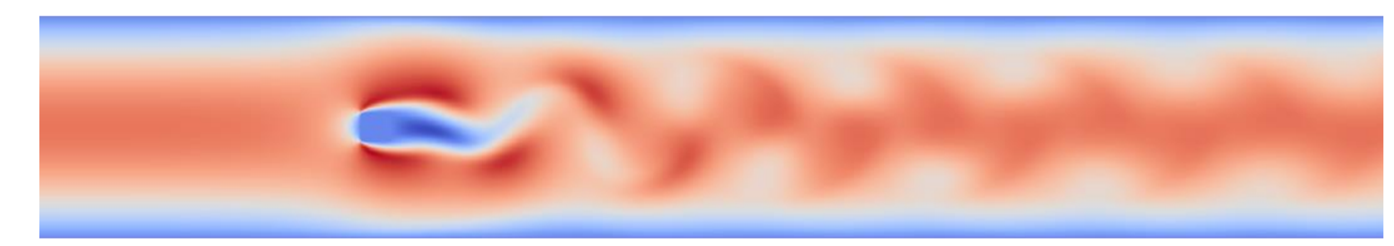
RESULTS (CONT'D)

- The drag coefficient values for different Reynolds number using SHSLBM and comparison with the traditional LBM in Ref. [1]

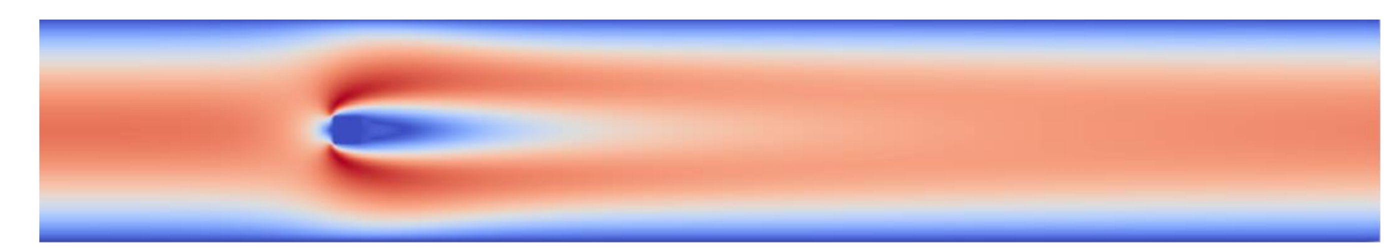
S. No.	Reynolds Number	Drag coefficient (C_d)	Drag coefficient (C_{dl}) Ref[1]
1	30	2.05	2.10
2	40	1.81	1.8
3	100	1.40	1.39

- The recirculation length comparison between SHSLBM and Ref. [1]

S. No.	Reynolds Number	L_r/D	L_r/D Ref. [1]
1	30	1.53	1.55
2	40	2.05	2.1



The velocity contour for Reynolds number 100



The velocity contour for Reynolds number 40

CONCLUSIONS

- The SHSLBM solver can solve the flow around the cylinder case accurately. This is illustrated from the comparison of the drag and the recirculation length to the previously cited values in Ref.[1].
- Moreover, the final solution obtained using SHSLBM depends a lot on the initialization of the domain. Proper domain initialization results in early removal of the noise in the simulation results. Thus, allowing to obtain the results faster.

Reference

- M. Breuer, J. Bernsdorf, T. Zeiser, F. Durst, Accurate computations of the laminar flow past a square cylinder based on two different methods: lattice-Boltzmann and finite-volume, International Journal of Heat and Fluid Flow, Volume 21, Issue 2, 2000, Pages 186-196, ISSN 0142-727X, https://doi.org/10.1016/S0142-727X(99)00081-8.

NOMENCLATURE

D: side of the square cylinder
C: speed of sound, in lattice units
e_α: lattice velocity direction
f_α: particle distribution function
f_α^{eq}: equilibrium distribution function
L_r: recirculation length
ρ: density
ρ^*: intermediate density
N: Number of directions
u: velocity
u^*: intermediate velocity
r: position vector
δ_t: time step
C_d: drag coefficient, present
C_{dl}: drag coefficient, literature
t: time
τ: relaxation parameter
μ: dynamic viscosity
ν: kinematic viscosity

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