hp-Adaptive Curvilinear Discontinuous Galerkin Spectral Element Method Acoustic Wave Equation Solver

Amit Nayak and Catherine Mavriplis

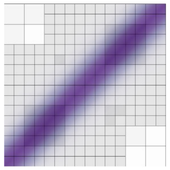
Department of Mechanical Engineering, University of Ottawa

Background

A hp-adaptive discontinuous Galerkin spectral element method solver with dynamic load balancing has been created for the acoustic wave equation [1]:

$$\frac{\partial^2 p}{\partial t^2} + c^2\left((u_x)_t + (v_y)_t\right) = 0$$

This solver currently models rectangular domains:

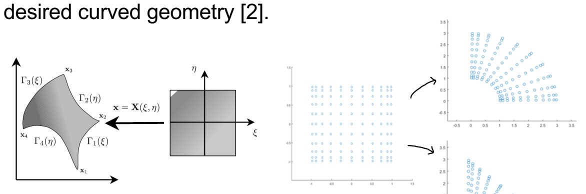


[1]

The objective of this work is to extend this solver to model curved domains, while maintaining hp-adaptivity.

Methodology

A geometrical mapping is made from a reference square to the desired curved geometry [2].



[2]

Mapping:

$$X(\xi,\eta) = \frac{1}{2}[(1-\xi)\Gamma_4(\eta) + (1+\xi)\Gamma_2(\eta) + (1-\eta)\Gamma_1(\xi) + (1+\eta)\Gamma_3(\xi)]$$
$$- \frac{1}{4}[(1-\xi)\{(1-\eta)\Gamma_1(-1) + (1+\eta)\Gamma_3(-1)\} + (1+\xi)\{(1-\eta)\Gamma_1(1) + (1+\eta)\Gamma_3(1)\}]$$

Derivative of Mapping:

$$\frac{\partial X}{\partial \xi} = \frac{1}{2}[\Gamma_2(\eta) - \Gamma_4(\eta) + (1-\eta)\Gamma_1{\prime}(\xi) + (1+\eta)\Gamma_3{\prime}(\xi)]$$
$$- \frac{1}{4}\{(1-\eta)[\Gamma_1(1) - \Gamma_1(-1)] + (1+\eta)[\Gamma_3(1) - \Gamma_3(-1)]\}$$

$$\frac{\partial X}{\partial \eta} = \frac{1}{2}[(1-\xi)\Gamma_4{\prime}(\eta) + (1+\xi)\Gamma_2{\prime}(\eta) + \Gamma_3(\xi) - \Gamma_1(\xi)]$$
$$- \frac{1}{4}\{(1-\xi)[\Gamma_3(-1) - \Gamma_1(-1)] + (1+\xi)[\Gamma_3(1) - \Gamma_1(-1)]\}$$

Jacobian:

$$J = a_1 \cdot (a_2 \times a_3) = X_\xi Y_\eta - Y_\xi X_\eta$$

Normals:

$$\hat{n}^1 = \frac{|J|}{J}\frac{Y_\eta \hat{x} - X_\eta \hat{y}}{\sqrt{Y_\eta^2 + X_\eta^2}}$$

$$\hat{n}^2 = \frac{|J|}{J}\frac{-Y_\xi \hat{x} + X_\xi \hat{y}}{\sqrt{Y_\xi^2 + X_\xi^2}}$$

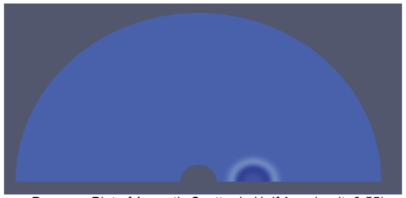
Scaling Factors:

$$scal_1 = \sqrt{Y_\eta^2 + X_\eta^2}$$

$$scal_2 = \sqrt{Y_\xi^2 + X_\xi^2}$$

Results

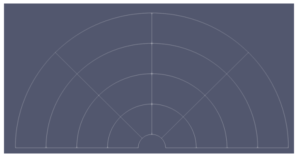
Pressure Plot of Acoustic Scatter in Half Annulus (t=0)



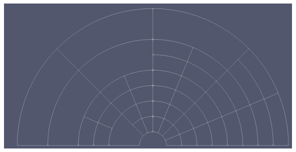
Pressure Plot of Acoustic Scatter in Half Annulus (t=0.55)

Pressure Plot of Acoustic Scatter in Half Annulus (t=3)

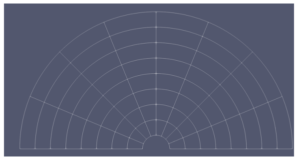
h-Adaptivity



K = 16 elements (t=0)



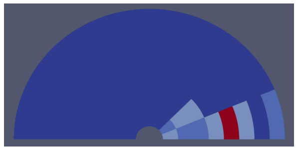
K = 46 elements (t=0.55)



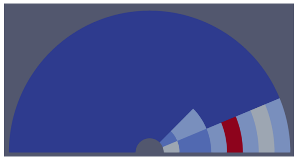
K = 64 elements (t=3)

p-Adaptivity

p = 10 (t=0)



p_{min} = 10, p_{max} = 24 (t=0.55)



p_{min} = 10, p_{max} = 24 (t=3)

Conclusions

A curvilinear geometry interface was created for the existing solver. hp-adaptivity was used to improve the quality of the resolution.

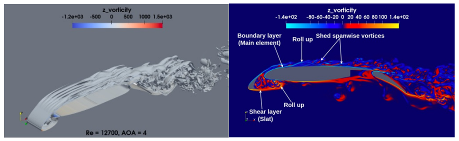
A simulation of an acoustic scattering in a half annulus was used to test the extension of the solver.

Future work includes testing different curved geometry cases and enabling dynamic load balancing.

Significance

Modelling curved geometries is advantageous as aerospace engineering problems rarely involve purely rectangular geometries.

This work contributes to direct numerical simulations in aerodynamics.



[3]

References

[1] S. He, "Dynamic Load Balancing for a hp-Adaptive Discontinuous Galerkin Wave Equation Solver via Spacing-Filling Curve and Advanced Data Structure," Master's thesis, University of Ottawa, 2021.

[2] D. Kopriva, Implementing Spectral Methods for Partial Differential Equations. Dordrecht: Springer, 2009.

[3] M. Vadsola, G. Agbaglah, and C. Mavriplis, "Slat Cove Dynamics of Low Reynolds Number Flow Past a 30P30N High Lift Configuration," Physics of Fluids, 33(3), 033607, 2021.

Nomenclature

- p: pressure
- u, v: velocity components
- c: wave speed
- t: time
- Γ: curved boundary
- ξ, η: computational space coordinates
- X: mapping
- K: number of elements
- p: polynomial order

Acknowledgements

CRSNG NSERC · FONCER · compute • calcul CANADA · Calcul Québec