Viscoplastic flows in superhydrophobic channels

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Introduction

Motivations:

- Superhydrophobic (SH) surfaces are used in microfluidic devices to manipulate the flow [1].
- Remarkable drag reduction has been demonstrated in channels with SH walls [1].
- In many applications, the fluid flowing in channels with SH walls shows viscoplastic rheology [2,3].

Applications in microfluidic systems:

- Human blood flow handling and manipulating in microchannels with SH walls, e.g. for disease diagnostics [2,3].
- Circulating tumor cells detection in microfluidic devices [2,3].

Applications in viscoplastic fluid flow transport:

- Transport of viscoplastic fluids, e.g. waxy crude oil, food and cosmetic products, and so on, in ducts and pipes in petroleum, food and pharmaceutical industries [4].

Viscoplasticity and superhydrophobicity:

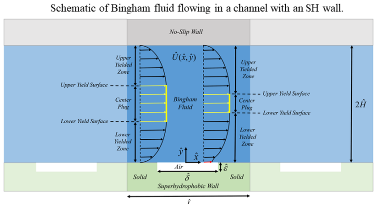
Bingham fluid flow through a channel with an SH wall is studied.

Bingham model

$$\begin{cases}\dot{\hat{\gamma}}=0, & \hat{\tau}\le\hat{\tau}_0\\ \hat{\tau}=\left(\dfrac{\hat{\tau}_0}{\dot{\hat{\gamma}}}+\hat{\mu}\right)\dot{\hat{\gamma}}, & \hat{\tau}>\hat{\tau}_0\end{cases}$$

$$\dot{\hat{\gamma}}=\left[\frac{1}{2}\dot{\hat{\gamma}}_{ij}\dot{\hat{\gamma}}_{ij}\right]^{\frac{1}{2}},\quad \hat{\tau}=\left[\frac{1}{2}\hat{\tau}_{ij}\hat{\tau}_{ij}\right]^{\frac{1}{2}}$$

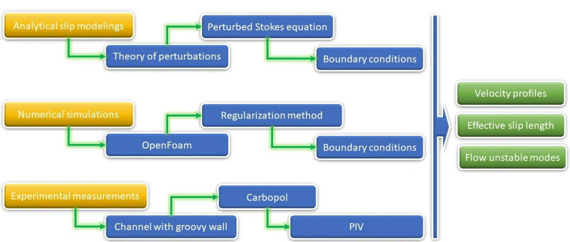
Schematic of Bingham fluid flowing in a channel with an SH wall.



Literature Review

Characterization of slip					
Experimental measurements		Mathematical modelings		Numerical simulations	
Newtonian	**Viscoplastic**	**Newtonian**	**Viscoplastic**	**Newtonian**	**Viscoplastic**
Choi, 2003	Vayssade, 2014	Philip, 1972	Panaseti, 2017a	Cheng, 2009	Damianou, 2016
Govardhan, 2009	Aktas, 2014	Vinogradova, 1995	Panaseti, 2017b	Davies, 2006	Damianou, 2014
Zhu, 2001	Daneshi, 2019	Belyaev, 2010	Ferras, 2012	Ou, 2005	
Hervet, 2003		Schmieschek, 2012	Kalyon, 2012	Priezjev, 2005	
Cheng, 2002		Asmolov, 2013		Asmolov 2013	**Shear-thinning**
Joseph, 2005				Schmieschek, 2012	Patlazhan, 2017
Tsai, 2009					Haase, 2017

Objectives



Governing Equations

Continuity Equation: $\nabla . U = 0$

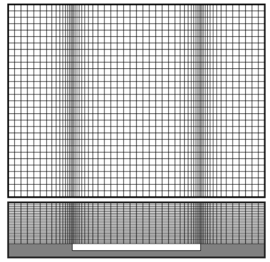
Navier-Stokes Equation: $R\left[\frac{\partial U}{\partial t}+(U.\nabla)U\right]=-\nabla P+\nabla.\tau$

Bingham model:
$$\begin{cases}\tau=\left(1+\dfrac{B}{\dot{\gamma}}\right)\dot{\gamma}, & \tau>B, \quad \tau=\sqrt{\tau_{ij}\tau_{ij}/2}\\ \dot{\gamma}=0, & \tau\le B, \quad \dot{\gamma}=\sqrt{\dot{\gamma}_{ij}\dot{\gamma}_{ij}/2}\end{cases}$$

Navier slip law (liquid/air interface): $u_s=b\left(1+\frac{B}{\dot{\gamma}_{la}}\right)\dot{\gamma}_{xy}$

Parameter	Name	Definition
R	Reynolds numbers	$\hat{\rho}\hat{U}_{ave}\hat{H}/\hat{\mu}_p$
B	Bingham number	$\hat{\tau}_0\hat{H}/\hat{\mu}_p\hat{U}_{ave}$
b	Slip number	$\hat{b}\hat{\mu}_p/\hat{H}$
ℓ	Groove periodicity length	$\hat{L}/\hat{H}$
φ	Slip area fraction	$\hat{\delta}/\hat{L}$

Numerical Simulation (NS)



- OpenFOAM 2.2.2 ⟶ Finite volume method
- SIMPLE algorithm ⟶ simpleFoam solver
- Steady-state condition ⟶ Residual of the velocity field < 10^{-9}
- Viscoplastic modeling ⟶ Papanastasiou regularization $\hat{\mu}_g=\hat{\mu}_p+\frac{\hat{\tau}_0\left(1-\exp(-m\dot{\hat{\gamma}})\right)}{\dot{\hat{\gamma}}}$.

$\ell=0.2,\quad \varphi=0.5$

$-\ell/2\le x\le\ell/2$

Semi-analytical modeling

Perturbing the equations of motion about the no-slip profile & using the Fourier transform [1]

$$\frac{\partial^4\hat{\psi}}{\partial y^4}-\left[\left(2+\frac{4B}{C_1+2C_2y}\right)n^2\kappa^2+in\kappa R\left(C_1y+C_2y^2\right)\right]\frac{\partial^2\hat{\psi}}{\partial y^2}+8Bn^2\kappa^2\frac{C_2}{(C_1+2C_2y)^2}\frac{\partial\hat{\psi}}{\partial y}+\left[n^4\kappa^4+in^3\kappa^3R\left(C_1y+C_2y^2\right)+i2n\kappa C_2R\right]\hat{\psi}=0$$

$$\hat{\psi}(n,0)=0,\quad \hat{\psi}(n,h)=0,\quad \frac{\partial\hat{\psi}}{\partial y}(n,h)=0,\quad \frac{\partial^2\hat{\psi}}{\partial y^2}(n,h)=0.$$

↓

$$u(x,y)=A_0\left(1-\frac{y}{h}\right)+\sum_{n=1}^{\infty}A_n^r\left[f_n'(y)\sin(n\kappa x)+g_n'(y)\cos(n\kappa x)\right]+\sum_{n=1}^{\infty}A_n^i\left[g_n'(y)\sin(n\kappa x)-f_n'(y)\cos(n\kappa x)\right]$$

$$v(x,y)=\sum_{n=1}^{\infty}A_n^r n\kappa\left[g_n(y)\sin(n\kappa x)-f_n(y)\cos(n\kappa x)\right]-\sum_{n=1}^{\infty}A_n^i n\kappa\left[g_n(y)\cos(n\kappa x)+f_n(y)\sin(n\kappa x)\right]$$

$$u_o(x,0)-b\left(\frac{\partial u_o}{\partial y}\right)_{y=0}=0,\quad 0\le x\le\frac{\varphi\ell}{2},\qquad u_o(x,0)=0,\quad \frac{\varphi\ell}{2}\le x\le\frac{\ell}{2}$$

⟶

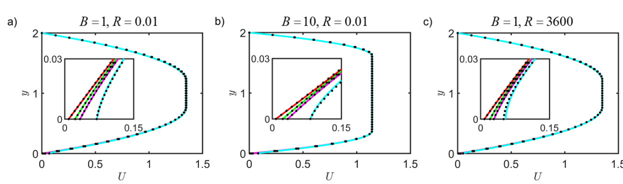
$$u_e(x,0)-b\left(B+\frac{\partial U_0}{\partial y}+\frac{\partial u_e}{\partial y}\right)_{y=0}=0,\quad 0\le x\le\frac{\varphi\ell}{2},\qquad u_e(x,0)=0,\quad \frac{\varphi\ell}{2}\le x\le\frac{\ell}{2}$$

↓

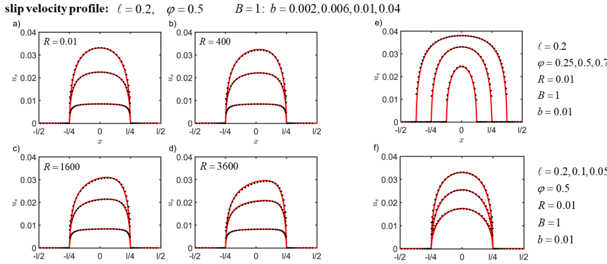
$$A_m^i=\sum_{n=1}^{N}\left[\left(P^i\right)^{-1}P^r\right]_{m,n}A_n^r\qquad\qquad \sum_{n=0}^{N}P_{mn}A_n=M_m$$

Results: Velocity profiles

Total velocity profile: $\ell=0.2,\quad\varphi=0.5$ — $B=1$: $b=0.002,0.006,0.01,0.04$ — $B=10$: $b=0.0005,0.0015,0.0023,0.007$

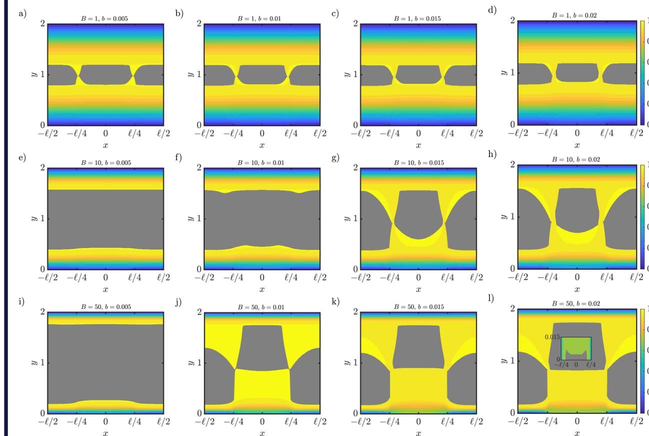


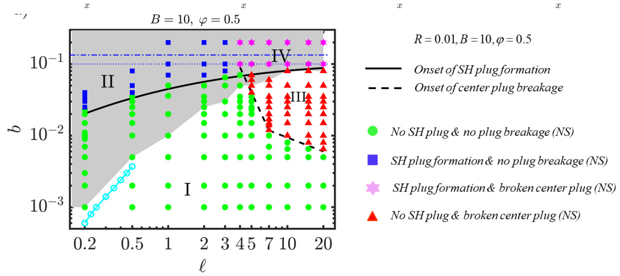
slip velocity profile: $\ell=0.2,\quad\varphi=0.5$ — $B=1$: $b=0.002,0.006,0.01,0.04$



Results: flow regimes

Towards SH wall plug formation & center plug breakage [2] — $R=0.01,\ \ell=7,\varphi=0.5$





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[1] H. Rahmani and S. M. Taghavi, "Poiseuille flow of a Bingham fluid in a channel with a superhydrophobic groovy wall," *J. Fluid. Mech.*, 2022.

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[3] T. A., Burinaru *et al.*, "Detection of circulating tumor cells using microfluidics," *ACS. combinatorial. Sci.*, 2018.

[4] A. O. Ijaola, P. K. Farayibi and E. Asmatulu, "Superhydrophobic coatings for steel pipeline protection in oil and gas industries: a comprehensive review," *J. Nat Gas Sci. & Eng.*, 2020.

Nomenclature

$\hat{\rho}$	Bingham fluid density, kg/m^3	$\dot{\hat{\gamma}}$	Shear rate, 1/s
$\hat{\mu}_p$	Bingham fluid plastic viscosity, Pa.s	$\hat{\tau}_0$	Yield stress, Pa
$\hat{U}_{ave}$	Flow average velocity, m/s	$\hat{H}$	Half channel height, m
$\hat{L}$	Groove period, m	$\hat{\delta}$	Slip region length, m
$\hat{m}$	Regularization parameter, s		

Acknowledgments

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