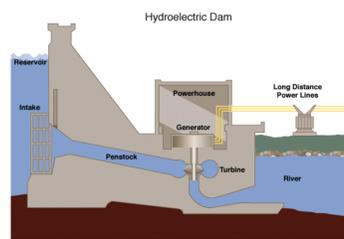
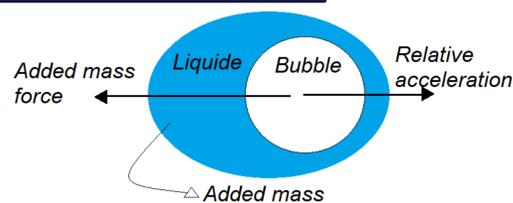


## AERATING TURBINES

- The water pumped from the reservoir to the river is often low in dissolved oxygen. The low level of oxygen is harmful to the aquatic life.
- Using aerating turbines is a solution to reduce drastically this impact.
- The aeration process involves injecting air bubbles at the turbine level.
- To develop more efficient and ecological turbines, the mechanics of bubble clouds are essential.



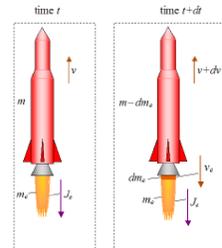
## The added mass force



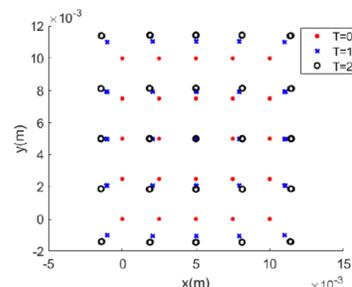
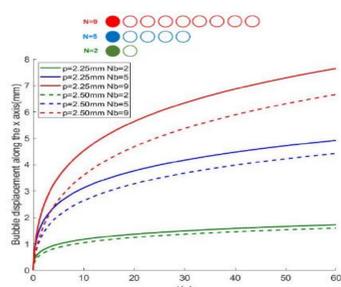
- For two bubbles rising side-by-side, the effects of added mass and induced added mass drive bubbles away from each other.
- The fact that added mass increases as bubbles come closer to each other, create a repelling force.

## LAGRANGIEN MODELISATION

- The added mass fluctuation is considered as a variable mass problem.
- A **“Meshchersky”** force appears from the spatial variation of the added mass.



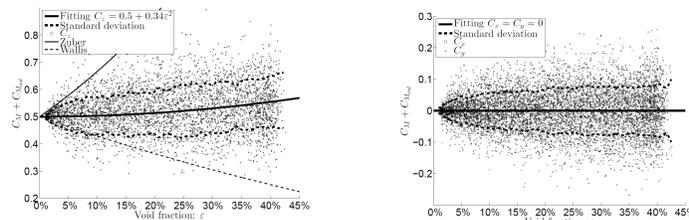
## The dispersive effect of the “Meshchersky” force



## EULERIAN MODELISATION

- Béguin et al. (2016) uses solid harmonics to solve 3D potential flow around bubbles.

### Added mass of the central bubble in a random cloud [Béguin et al. (2016)]



- Actual added mass models do not consider local variations of the added mass due to local bubble distribution.
- The dispersion effect of the “Meshchersky” force cannot be captured.
- Objective** : To develop a new added mass model for the Eulerian approach that considers local bubble configuration by introducing the void fraction gradient.

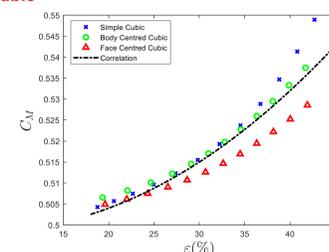
$$\hat{C}_M = f(\varepsilon, \nabla \varepsilon)$$

## SYMMETRICAL CONFIGURATIONS



- Added mass Correlation:**

$$C_M = \frac{1}{2} - 0.04\varepsilon + 0.30\varepsilon^2$$



## ASYMMETRICAL CONFIGURATIONS

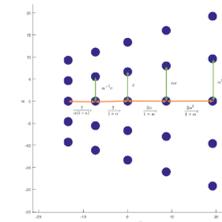
- Uniform transformation for the regular configurations to create a void fraction gradient around the central bubble :

### Transformation of the cubic structure

$$x_t = \frac{2c\alpha}{1+\alpha} \frac{1-\alpha}{1-\alpha} \frac{x_{reg}}{c}$$

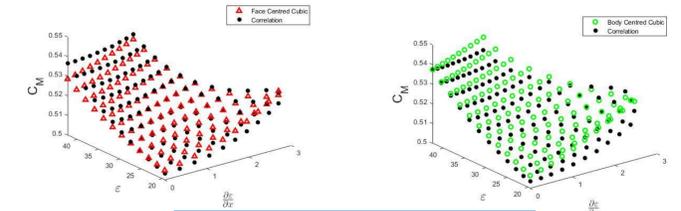
$$y_t = y_{reg} \alpha \frac{x_{reg}}{c}$$

$$z_t = z_{reg} \alpha \frac{x_{reg}}{c}$$



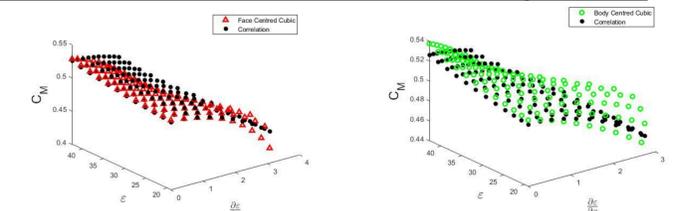
## ADDED MASS CORRELATION

- Acceleration perpendicular to the void fraction gradient :**



$$C_{xx} = \frac{1}{2} - 0.04\varepsilon + 0.30\varepsilon^2 + 24\nabla \varepsilon_{xx}^2$$

- Acceleration parallel to the void fraction gradient :**



$$C_{zz} = \frac{1}{2} - 0.04\varepsilon + 0.30\varepsilon^2 - 68\nabla \varepsilon_{zz}^2$$

- $C_{MT}$  and  $C_{MN}$  are respectively the tangential and normal components of the added mass tensor to the acceleration.
- $\alpha$  is the angle between the acceleration and the void fraction gradient :

$$C_{MT} = C_{xx} \sin \alpha$$

$$C_{MN} = C_{zz} \cos \alpha$$

## CONCLUSIONS

- Including the void fraction gradient in the added mass modeling, allows a better representation of its dependence on bubble cloud configuration.
- The fluctuation of the added mass results in a **“Meshchersky”** force that should be included in numerical simulations to model small bubble dispersion.

## FURTHER WORK

- Validate correlations with random bubble cloud configurations.
- Develop a **“Meshchersky”** force model for Euler-Euler approach.
- Include the new added mass model in a numerical CFD code to simulate bubble column and transverse bubble jet experiments.

## ACKNOWLEDGMENTS