Whispering Gallery Modes Intrinsic Quality Factor and Coupling Regime Extraction Using Stokes Parameters

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***Abstract*—We propose a novel method to characterize Whispering Gallery Modes and ring-type optical resonators. Based on a Stokes parameters analysis, the intrinsic Q-factor and the coupling Q-factor can be distinctly identified for any coupling regime.**

***Keywords*—Whispering Gallery Mode, Optical microcavity, Polarization measurements, Intrinsic Quality Factor, Coupling regime.**

I. Introduction

Whispering Gallery Modes (WGM) and ring-type optical resonators have already shown great potential in many domains such as telecommunications and biosensing [1], [2]. Optimization of the performance depends on the ability to fully characterize WGM losses which affect the spectral transmission of the signal. In a waveguide-cavity configuration, the total quality factor (Q_T) of a resonance characterizes the combined losses caused by the intrinsic losses (Q_0) due to scattering and absorption and the coupling losses due the waveguide (Q_c). Depending on the dominant losses source, the system is overcoupled $Q_c < Q_0$, undercoupled $Q_c > Q_0$ or critically coupled $Q_c = Q_0$.

The simplest way to measure Q_T is using the full width half maximum ($\Delta\lambda$) of the resonant peak in the stationary regime. A single measurement of Q_T is not sufficient to discriminate between the losses caused either by the intrinsic absorption and scattering or due to coupling, unless critical coupling is achieved ($Q_0 = Q_c = 2Q_T$). A simple model can be used to find Q_0 and Q_c based on the transmission spectra if the coupling regime is first known [3]. This can be problematic for a fixed waveguide setting such as integrated microrings, since the gap between the cavity and the waveguide cannot be varied to verify the coupling regime.

The importance of optimizing Q_0 has been pointed out for optical devices [4], [5] and its determination would help optimize microfabrication processes and reduce losses. Previous work based on a fast laser sweeping across the cavity resonance showed it is possible to retrieve Q_0 and Q_c using a numerical fit [6]. Unfortunately, this method is difficult to apply to cavities with a quality factor below 10^7 since the laser sweeping speed needed for such an operation becomes too high (> 500 MHz/μs) [6].

In this communication, we propose a single measurement method to extract Q_0 and Q_c using a model based on a Stokes parameters analysis. This method is valid for Q_T above 10^5 regardless of the coupling regime and the input polarization state. The experimental results obtained are in good agreement with the theoretical model.

II. Stokes Parameters Analysis

Considering the normalized input amplitudes $\vec{a}_x + \vec{a}_y = a_x\vec{x} + a_y e^{i\phi}\vec{y}$ where ϕ is the phase difference between $\vec{a}_x$ and $\vec{a}_y$, the Stokes parameters can be expressed in terms of the cavity parameters T and θ and the system parameters a_x, a_y and ϕ as follows [7], [8]:

$$\begin{aligned} S_0 &= a_x^2 + |T|^2 a_y^2 \\ S_1 &= a_x^2 - |T|^2 a_y^2 \\ S_2 &= 2a_x a_y |T| \cos(\theta + \phi) \\ S_3 &= 2a_x a_y |T| \sin(\theta + \phi). \end{aligned} \quad (1)$$

In this case, $\vec{x}$ and $\vec{y}$ are the TE and TM axis of the whispering gallery mode and only the TM mode resonates in the considered wavelength range. $|T|^2$ and θ are part of the complex response of the cavity $|T| \exp(i\theta)$ [9]:

$$\begin{aligned} |T|^2 &= \frac{t^2 + e^{-2\alpha L} - 2te^{-\alpha L}\cos(\beta_0 L)}{1 + t^2 e^{-2\alpha L} - 2te^{-\alpha L}\cos(\beta_0 L)} \\ \theta &= \tan^{-1}\left(\frac{[t^2-1]e^{-\alpha L}\sin(\beta_0 L)}{t(1+e^{-2\alpha L}) - e^{-\alpha L}[1+t^2]\cos(\beta_0 L)}\right) \end{aligned} \quad (2)$$

where β_0 and L are the wavenumber and the round-trip path of the mode inside the cavity respectively. Finally, α and t are related to Q_0 and Q_c by $Q_0 = \beta_0/2\alpha$ and $Q_c = \beta_0 L/(1-t^2)$. Figure 1 shows S_0 (blue), S_3 with $\phi = 0$ (red) and S_3 with $\phi = -\pi/3$ (green) theoretical spectrum for (a) undercoupled regime (b) critically coupled regime and (c) overcoupled regime. As it can be seen, the S_3 spectra amplitude $\Delta S_3 = S_3^{max} - S_3^{min}$ is increasing as the coupling losses increase (Q_c decreases). Additionnaly, ΔS_3 remains constant if ϕ is changed for a fixed coupling regime. Compared to the regular transmission spectra where no information about coupling regime is given, the ΔS_3 parameter gives the

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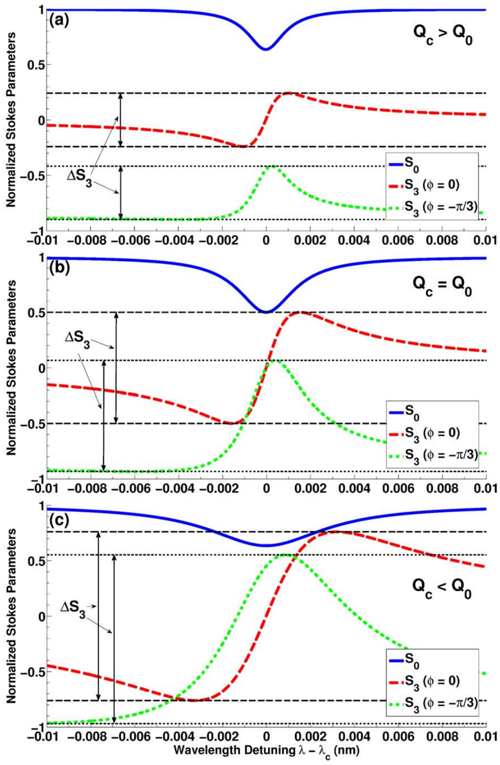

Fig. 1. Normalized Stokes parameters theoretical spectra of S_0 (blue), S_3 with $\phi = 0$ (red) and S_3 with $\phi = -\pi/3$ (green) as a function of the resonance wavelength λ_c detuning for (a) undercoupled regime, (b) critically coupled regime and (c) overcoupled regime. The dotted black lines show the maximum S_3^{max} and the minimum S_3^{min} of S_3 curves.

coupling regime: undercoupled if $\Delta S_3 < 1$, critically coupled if $\Delta S_3 = 1$ and overcoupled if $\Delta S_3 > 1$.

III. Q_0 and Q_c Extraction

The ΔS_3 parameter can also be used to extract Q_0 and Q_c. Relating Q_T and ΔS_3 to Q_0 and Q_c with

$$Q_T = \frac{\lambda}{\Delta\lambda} = \frac{Q_0 Q_c}{Q_0 + Q_c} \text{ and } \frac{\Delta S_3}{2a_x a_y} \approx \frac{2Q_0 - \beta_0 L}{Q_0 + Q_c - \beta_0 L}, \quad (3)$$

it is possible to find that Q_0 and Q_c can be approximated by

$$Q_0^{(e)} = \frac{4a_x a_y Q_T}{4a_x a_y - \Delta S_3} \text{ and } Q_c^{(e)} = 4a_x a_y \frac{Q_T}{\Delta S_3} \quad (4)$$

when $\beta_0 L$ can be neglected compared to Q_0 and Q_c. These equations show that, knowing a_x and a_y, a single measurement of the FWHM ($\Delta\lambda$) and ΔS_3 gives $Q_c^{(e)}$ and $Q_0^{(e)}$, assuming Q_0 and Q_c high enough, which is the case generally considering WGM resonator types cavities. Additionally, the fact that Q_0 and Q_c are independent of ϕ implies an insensitivity to phase fluctuations.

Considering the relative error caused by the approximation used, it is found that the relative error $(Q_0^{(e)} - Q_0)/Q_0$ is smaller than 1% if $Q_0 \geq 10^4$ and $Q_c \geq 10^5$. The relative error on $Q_c^{(e)}$ is also smaller than 1% if $(Q_0, Q_c) \geq (10^4, 7.5 \times 10^4)$ or $(9 \times 10^4, 10^4)$. Following the Q_T definition in Eq. 3, measuring a $Q_T > 10^5$ gives a theoretical error below 1%.

Figure 2 shows a typical experimental curve of S_0 in green and S_3 in blue taken with a microtoroid cavity. The extracted $Q_0^{(e)}$ and $Q_c^{(e)}$ are $(2.69 \pm 0.07) \times 10^6$ and $(1.73 \pm 0.15) \times 10^7$ respectively. Using these values in the model (Eq. 1 and Eq. 2) gives the theoretical S_0 (black line) and S_3 (red line) using a power ratio of $a_y^2/a_x^2 = 0.87$. ϕ is obtained from the off-resonance value of the S_3 curve. The good agreement between both curves indicates that the proposed model represents well the polarization changes in cavity.

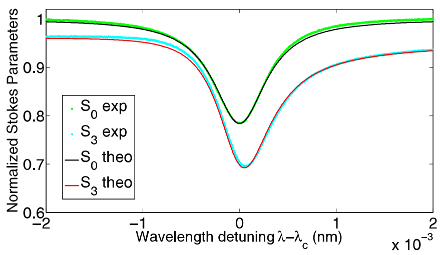

Fig. 2. Experimental (green and blue) and theoritical (black and red) curves of S_0 and S_3. The extracted quality factors are $Q_0^{(e)} = (2.69 \pm 0.07) \times 10^6$ and $(1.73 \pm 0.15) \times 10^7$.

IV. Conclusion

We proposed a single spectral scan method, based on a Stokes parameters analysis, to extract Q_0 and Q_c from a WGM cavity-waveguide coupled system. We defined the lowest measurable Q_T needed in order to estimate Q_0 and Q_c within a 1% error range. Once the extracted Q_0 and Q_c are inserted in the model, the theoretical curves and experimental spectrum are in good agreement.

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