Intrinsic quality factor determination in whispering gallery mode microcavities using a single Stokes parameters measurement

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Abstract: Determination of the intrinsic quality factor of a loaded whispering gallery mode microcavity can be important for many applications where the coupling conditions cannot be tuned. We propose a single-scan technique based on a Stokes parameters analysis to extract the intrinsic quality factor and therefore determine the coupling regime. We propose a simple model for this analysis and present experimental measurements, which are in very good agreement with the model.

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References and links

1. Introduction

Whispering gallery mode (WGM) optical microcavities are widely known and used in many applications such as single molecule detection [1], nanoparticle sizing [2], compact narrow band filters [3], microlasers [4], optomechanical systems [5] and light switching [6]. Several materials are used in all these applications: Si, doped and undoped SiO$_2$, fluoride materials, III-V and II-VI semiconductors, polymers, chalcogenide glass, etc. WGM microcavities exist mostly in the form of microspheres [7], microdisks [4], microtoroids [8] and integrated ring resonators [9]. Main features of these microcavities include their high intrinsic quality factor ($Q_0$), their compactness and for some, their ease for on-chip integration.

Their optical characterization is usually done from their radiative emission spectrum collected by a free space setup or by measuring the transmission spectrum using the evanescent coupling to a waveguide. In the former case, the mode $Q_0$, related to the losses caused by scattering, absorption and radiative process, is directly obtained from the resonance peak full width half maximum (FWHM or $\Delta \lambda$). In the latter case, $\Delta \lambda$ is linked to the loaded or total quality factor ($Q_T = \lambda / \Delta \lambda$). It is related to $Q_0$ and the coupling quality factor $Q_c$ by $Q_T^{-1} = Q_0^{-1} + Q_c^{-1}$. As the coupling is increased, the light tends to leave the cavity sooner via the waveguide, thus increasing the losses due to the coupling and decreasing $Q_c$. Depending on the dominating loss processes, the coupling regime can be identified as undercoupled ($Q_0 < Q_c$), critically coupled ($Q_0 = Q_c = 2Q_T$) or overcoupled ($Q_0 > Q_c$). Knowing precisely the contribution of the intrinsic losses and the coupling losses can be important in order to optimize the fabrication process or for practical applications. For example, for sensing applications, the optimum sensitivity is achieved using a $Q_c = 2Q_0$ configuration [10]. In the case of filters or dispersion compensation devices, the maximization of $Q_0$ is desirable [11].

The simplest approach to measure $Q_0$ and $Q_c$ is done by using $Q_T$ and the normalized transmission value at the resonance wavelength $\lambda_r$ [12]. Unfortunately, due to their symmetric contribution to $Q_T$, the coupling regime has to be known to distinguish between the $Q_0$ and $Q_c$ contributions, except when critically coupled where the normalized transmission is zero. The coupling regime can be identified by varying the gap between the waveguide and the cavity. Since sensors or telecommunications devices are mainly designed to be on-chip with fixed waveguide configuration, a correct determination can be difficult. Most Fabry-Perot characterization techniques cannot be used since a modulated signal in the frequency range of the free spectral range (FSR) is needed [13, 14]. Since WGM cavities usually have FSR above 100 GHz, instruments operating at these frequencies are not only very expensive, but they make the experimental setup bulky and cumbersome as well. Dumeige et al. [15] showed that $Q_0$ and $Q_c$ extraction is possible if a laser line is swept fast enough across the resonance, because the
resulting ringing phenomenon does not depend on $Q_0$ and $Q_c$ in the same way. Unfortunately, this technique requires a high speed sweep and thus, cannot be applied to $Q_T$ below $10^7$. Finally, Ito et al. [16] also proposed a method limited to the case of two busline waveguides using the drop port and throughput port responses where both waveguides have the same coupling coefficient.

In this article, we propose a simple method to extract both $Q_0$ and $Q_c$ and therefore the coupling regime of WGM and ring-type cavities. Based on a Stokes parameters analysis near the resonance wavelength, this single scan technique can be used to determine $Q_0$ and $Q_c$ when $Q_T$ is above $1 \times 10^5$ with $<1\%$ estimation error. Also, the method does not depend on the coupling regime nor on the input polarisation states. First, the theoretical model used to describe the Stokes parameters analysis is explained for different coupling regimes. The estimation of both quality factors is described with simple relations and the minimum $Q_T$ where the technique is valid is determined based on the estimation errors. Then, we present the experimental setup and the measurements for different coupling gaps. The very good agreement of the experimental Stokes parameters curves supports the reliability of the proposed technique.

2. Theoretical model

The model used to extract $Q_0$ and $Q_c$ is based on a polarization analysis (Stokes parameters) where the phase change in the WGM cavity gives, along with $Q_T$, an additional information about the coupling regime. In this section, we present the theoretical basis of how $Q_0$ and $Q_c$ are determined and the conditions for which this technique is applicable.

The coupling arrangement is described in Fig. 1. An input polarization state $\vec{a}_x + \vec{a}_y = a_x \hat{x} + a_y e^{i\phi} \hat{y}$ enters the coupling region where $a_x$ and $a_y$ are the modulus of the $x$ and $y$ components, and $\phi$, their phase difference. Only one axis is coupled to the cavity mode, we chose $\vec{a}_y$ in this article.

Thus, the $x$ and $y$ axes correspond to the quasi-TE and quasi-TM modes of the cavity. The coupled ($\kappa^2$) and uncoupled ($t^2$) power fractions are related to each other as $\kappa^2 + t^2 = 1$ in the lossless case. They are related to $Q_c$ by $Q_c = k_0 n_m L / (1 - t^2)$ [17], where $k_0 = 2\pi/\lambda$, $n_m$ and $L$ are the vacuum wavenumber, the mode refractive index and the round-trip path of the mode in the cavity respectively. The losses inside the cavity are described by the attenuation factor $\alpha$ via $\exp(-\alpha L)$. When the losses are small ($\exp(-\alpha L) \approx 1 - \alpha L$), $\alpha$ is related to the intrinsic

![Fig. 1. Lossless coupling scheme between a single mode waveguide and an optical resonator. Polarization states $a_x \hat{x} + a_y e^{i\phi} \hat{y}$ and $b_x \hat{x} + b_y \hat{y}$ are entering and exiting the coupling region respectively. An amplitude transmission coefficient of $|T|$ and a phase difference of $\theta$ are added by the cavity. The power fraction $\kappa^2 = 1 - t^2$ coupled to the cavity and the losses $\alpha$ characterize the waveguide-resonator system.](image-url)
quality factor by \( Q_0 = k_0 n_m/(2\alpha) \). The exiting polarization state is described by \( \vec{b}_x + \vec{b}_y \) and it can be expressed using a Jones matrices formulation [18]:

\[
\begin{bmatrix}
 b_x \\
 b_y 
\end{bmatrix} =
\begin{bmatrix}
 1 & 0 \\
 0 & |T|e^{i\theta} 
\end{bmatrix}
\begin{bmatrix}
 a_x \\
 a_y e^{i\phi} 
\end{bmatrix} .
\]

We assume that the coupling region is lossless and the resonant frequencies of the TE/TM modes are sufficiently separated to leave \( \vec{a}_i \) unperturbed by the cavity presence. If a mode were not sufficiently TE or TM, it would imply that \( \vec{a}_i \) would be perturbed by the cavity response as well and this proposed model would no longer describe properly the polarization changes. We experimentally validated that the mode is quasi-TE(TM) by making sure that the measured resonance peak can be fully turned off (flat transmission) and back on by rotating the input linear polarization using the first half-wave plate HWP1 (see Fig. 3). The cavity’s Jones matrix represents the complex response of a cavity mode coupled to a waveguide mode as follows [17]:

\[
|T|^2 = \frac{r^2 + e^{-2\alpha L} - 2e^{-\alpha L}\cos(\beta_0 L)}{1 + r^2e^{-2\alpha L} - 2e^{-\alpha L}\cos(\beta_0 L)} \quad (1)
\]

\[
\theta = \tan^{-1}\left(\frac{|r^2 - 1|e^{-\alpha L}\sin(\beta_0 L)}{r(1 + e^{-2\alpha L}) - e^{-\alpha L}[1 + r^2]\cos(\beta_0 L)}\right) \quad (2)
\]

where \( \beta_0 = k_0 n_m \). \(|T|^2\) is the power transmission coefficient and \( \theta \) is the phase change due to the interaction with the cavity. We assume the cavity is passive (\( Q_0 > 0 \)) and that \( t, \kappa, \phi \) and \( \alpha \) are constant over the spectral range of interest, typically twice the FWHM.

The Stokes parameters can now be expressed in terms of the cavity parameters \( (T, \theta) \) and the system parameters \( (\alpha_x, \alpha_y, \phi) \) as follows [19]:

\[
S_0 = |b_x|^2 + |b_y|^2 = a_x^2 + |T|^2 a_y^2
\]

\[
S_1 = |b_x|^2 - |b_y|^2 = a_x^2 - |T|^2 a_y^2
\]

\[
S_2 = 2|b_x||b_y|\cos(\delta) = 2a_x a_y |T|\cos(\theta + \phi)
\]

\[
S_3 = 2|b_x||b_y|\sin(\delta) = 2a_x a_y |T|\sin(\theta + \phi)
\]

where \( \delta \) is the accumulated phase difference between axis. These expressions reveal that a spectral characterization of the Stokes parameters provides, along with the normalized transmission \( S_0 \), an additional information about the phase \( \theta \) considering a non-zero value of \( \alpha_x \) and \( \alpha_y \). Since \( S_0 \) and \( S_2 \) are similar to \( S_1 \) and \( S_3 \) respectively, the following equations are developed for \( S_2 \) but one can use \( S_3 \) as well since it only has a \( \pi/2 \) phase difference.

It is possible to represent \( S_2 \) as a function of the system parameters \( t, \alpha, L, \beta_0 \) and \( \phi \) by inserting Eq. (1) and Eq. (2) into the definition for \( S_2 \) using trigonometric identities:

\[
S_2 = 2a_x a_y \frac{\cos(\phi)[r(1 + e^{-2\alpha L}) - e^{-\alpha L}(1 + r^2)\cos(\beta_0 L)] + \sin(\phi)e^{-\alpha L}(1 - r^2)\sin(\beta_0 L)}{1 + r^2e^{-2\alpha L} - 2e^{-\alpha L}\cos(\beta_0 L)}. \quad (4)
\]

In Fig. 2, we present the Stokes parameters spectra calculated from Eq. (3) for different coupling regimes and different \( \phi \) values. The spectra of \( S_0 \) in blue, \( S_1 \) in purple, \( S_2 \) in green and \( S_3 \) in red are shown for each case. It can be seen that the \( S_1 \) does not provide additional information compared to \( S_0 \) since it is only its complementary response.

The increasing amplitude of the \( S_2 \) and \( S_3 \) spectra as \( Q_e \) decreases can be understood considering that more photons are entering the cavity and are then recollected by the waveguide. Consequently, the polarization state of a higher amount of light is changed by the cavity. Since the \( S_0 \) spectrum tends to flatten as \( Q_e \) decreases, the \( S_2 \) spectrum can be used to spot collapsed peaks, mainly in the overcoupled regime (Fig. 2(e) and 2(f)).
Fig. 2. Calculated spectra of the Stokes parameters across a resonance for the undercoupled regime 2(a)-2(b) \( Q_0 = 1 \times 10^6 \) and \( Q_c = 3 \times 10^6 \), the critically coupled regime 2(c)-2(d) \( Q_0 = Q_c = 1 \times 10^6 \) and the overcoupled regime 2(e)-2(f) \( Q_0 = 1 \times 10^6 \) and \( Q_c = 3 \times 10^6 \).

The black dots indicate the extrema of the \( S_3 \) curves and the red dots show the points that define the FWHM of the \( S_0 \) curves. For \( \phi = 0 \), the wavelength positions of the black and red dots match for any coupling regime. In the case where \( \phi \neq 0 \), the positions no longer agree and the distance between black dots increases as the coupling increases. However, it can be seen that \( \Delta S_2 = S_2^{\text{max}} - S_2^{\text{min}} \) and \( \Delta S_3 = S_3^{\text{max}} - S_3^{\text{min}} \) do not change when \( \phi \) is modified but increase monotonically when the coupling increases. Their values go from 0 to 1 as the coupling regime passes from undercoupled to critically coupled and from 1 to 2 as it passes from critically coupled to overcoupled. Compared to the resonance transmission value which can give the same value in the undercoupled regime and in overcoupled regime [12], \( \Delta S_2 \) or \( \Delta S_3 \) can be used to determine the coupling regime.

An additional feature of the \( S_3 \) spectrum (or \( S_2 \) spectrum depending on \( \phi \) value) is its steep slope near the resonance, which is a consequence of its phase response dependency (Eq. (3)). This attribute can be exploited in applications where high sensitivity is required, such as biosensing or laser stabilization.

Using a normalized \( S_2 \) definition, \( S_2^N = S_2/(2a_x a_y) \), it is possible to estimate \( Q_0 \) and \( Q_c \). This

\[ \Delta S_2 = S_2^{\text{max}} - S_2^{\text{min}} \]
\[ \Delta S_3 = S_3^{\text{max}} - S_3^{\text{min}} \]
Finally, both $Q_c$ and $\Delta Q_c$ can be estimated from Eqs. (5) and Eq. (6) as:

$$Q_c^{(e)} = \frac{4a_x a_y Q_T}{4a_x a_y - \Delta S_2} = \frac{2Q_T}{2 - \Delta S^N_2}$$

and

$$Q_c^{(e)} = 4a_x a_y \frac{Q_T}{\Delta S_2} = \frac{2Q_T}{\Delta S^N_2}$$

when $\beta_0 L$ can be neglected compared to $Q_0$ and $Q_c$. Thus, knowing $a_x$ and $a_y$, both $Q_c^{(e)}$ and $Q_0^{(e)}$ can be extracted using a single measurement of $\Delta \lambda$ and $\Delta S_2$. In order to be precise, these estimations require that the real $Q_0$ and $Q_c$ be high enough. This condition is usually achieved for WGM and ring-type resonators. Also, since both are independent of $\phi$, phase fluctuations do not change the results.

For a real measurement, however, we need to know within which limits these approximations hold based on a measurable quantity. We shall now define the lowest $Q_T$ value needed in order to estimate $Q_0$ and $Q_c$ within an acceptable error range. The relative errors of $Q_c$ and $Q_0$, $\Delta Q_c/Q_c$ and $\Delta Q_0/Q_0$, are calculated from $\Delta S_2$ and $Q_T$ for each $(Q_0, Q_c)$ combination, for $Q_0$ and $Q_c$ above $10^4$. We limit the analysis to $10^4$ since previous assumptions ($t$, $\kappa$, $\alpha$, and $\phi$ constant) are not guaranteed below this limit. This lower value of $Q_c$ implies a power coupling coefficient $k^2$ up to 0.18 which includes realistic values below 0.05 or $Q_c > 3.7 \times 10^4$ for a WGM cavity-to-waveguide evanescent coupling [20]. Finally, we set $a_x = a_y = 1/\sqrt{2}$ where $S_2$ amplitude is maximized.

Using the calculations above, the relative errors in $Q_0$ and $Q_c$ can be expressed as:

$$\frac{\Delta Q_0}{Q_0} = 1 - \frac{4a_x a_y Q_T}{(4a_x a_y - \Delta S_2)Q_0},$$

$$\frac{\Delta Q_c}{Q_c} = 1 - \frac{4a_x a_y Q_T}{Q_c \Delta S_2}.$$
For a relative error smaller than 1%, in the case of $Q_0^{(e)}$, it requires a $Q_0 \geq 10^4$ and a $Q_e \geq 10^5$. For $Q_0^{(o)}$, the combination $(Q_0, Q_e)$ must be higher than $(1 \times 10^4, 7.5 \times 10^4)$ or $(9 \times 10^4, 1 \times 10^4)$. From Eq. (6), for a given $Q_T$ value, $Q_0$ and $Q_e$ are higher than $Q_T$. Consequently, the measured $Q_T$ has to be above $10^5$ in order to be within the 1% error limit of $Q_0^{(e)}$ and $Q_0^{(o)}$. For an error below 5%, a measured $Q_T \geq 2 \times 10^4$ is required.

3. Measurements and discussion

In order to relate the detected output signal of the cavity and the Stokes parameters, simple optical analysis can be used. The intensity of a signal passing through a wave plate with a dephasing angle of $\phi_0$ and a polarizer rotated of $\alpha_p$ compared to the wave plate axis is [19]:

$$I(\alpha_p, \phi_0) = \frac{1}{2} \left[ S_0 + S_1 \cos(2\alpha_p) + S_2 \cos(\phi_0) \sin(2\alpha_p) + S_3 \sin(\phi_0) \sin(2\alpha_p) \right].$$

The Stokes parameters can be related to different configurations of this optical system as follows:

$$S_0 = I(0^\circ, 0^\circ) + I(90^\circ, 0^\circ)$$
$$S_1 = I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ)$$
$$S_2 = 2I(45^\circ, 0^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) = 2I(45^\circ, 0^\circ) - S_0$$
$$S_3 = 2I(45^\circ, 90^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) = 2I(45^\circ, 90^\circ) - S_0.$$

It should be noted that $S_2$ is easier to measure than $S_3$ since it does not need a quarter wave plate. $S_0$ and $S_2$ can be obtained by measuring the total power, $I(0^\circ, 0^\circ) + I(90^\circ, 0^\circ)$, and the intensity after the polarizer rotated by $45^\circ$ with respect to the chosen reference axis, $I(45^\circ, 0^\circ)$. The experimental setup used to simultaneously determine $S_0$ and $S_2$ is shown in Fig. 3. A tunable laser source (TLS) sweeps across the resonance wavelength while an oscilloscope (OSC) records the signal from the detectors D1 and D2. Both are synchronized via a trigger signal (TRIG). The first half wave plate HWP1 is used to rotate the linearly polarized output of the source and therefore, to control the input power ratio $a_x^2/a_y^2$. To ensure that the polarization state did not change along the SMF-28 fiber, the optical fiber length and curvature were minimized. Furthermore, before measurement and without any coupling to the cavity, the output polarization was measured after the collimator CO3 to ensure that it stayed linear along the fiber. To do so, a linear polarizer and a free space detector were used to verify the flatness of the polarization ellipse. We did not notice any significant change compared to the polarization ellipse measured.

![Fig. 3. Experimental setup used for $S_0$ and $S_2$ parameters measurements: TLS - Tunable laser source, OSC - Oscilloscope, TRIG - Trigger signal, CAV - Microcavity, TAP - Tapered fiber, C - 50:50 non-polarizing beamsplitter cube, CO1, CO2 et CO3 - Collimators, HWP1 et HWP2 - λ/2 wave plates, POL - Polarizer, L1 et L2 - Lens, D1 et D2 - Detectors. A SMF-28 fiber is used up to CO3.](image-url)
right after the collimator CO1. Thus, we conclude that the polarization along the optical fiber and the tapered fiber stays linear. The second half wave plate HWP2 is adjusted such as the $x$ and $y$ axis are turned $45^\circ$ compared to the horizontal and vertical reference axis. This allows to write the detector intensities as a function of the cavity parameters in a simple form (Eq. (7)). Finally, the polarizer axis is set along the horizontal axis or $45^\circ$ compared to $a_x$. Both signals are focused on the detectors. This simple configuration can be used to measure the transmission spectrum via D1 without any modification. Since both signals are recorded simultaneously, any fluctuation in the resonance wavelength due to external parameters such as temperature does not affect the result.

A 1.2 $\mu$m diameter tapered optical fiber is used to couple the light to a silica toroidal microcavity. The microcavities are formed from a 0.8 $\mu$m thick thermal SiO$_2$ layer. Using a standard photolithographic process, the disk shapes are transfered to the silica layer. An SF$_6$ isotropic etch of the subjacent silicon follows. The toroidal shapes are obtained using laser reflow process [8]. The microtoroids have a 5 $\mu$m minor diameter and are formed out of a 100 $\mu$m diameter disk. A micrograph of the coupling region is shown in Fig. 4. The gap between the tapered fiber and the cavity is controlled using a piezoelectric stage.

Using this experimental setup, the intensities detected at D1 and D2, $I_1$ and $I_2$, can be written as:

\begin{align}
I_1 & = |T_{45}|^2 a_x^2 + |T_{-45}|^2 |T|^2 a_y^2 \approx |T_{45}|^2 (a_x^2 + |T|^2 a_y^2) \\
I_2 & \approx \frac{p_x^2 |T_{45}|^2}{2} [a_x^2 + |T|^2 a_y^2 + 2 a_x a_y |T| \cos(\theta + \phi)] .
\end{align}

where the $|T_{45}|^2$ and $|T_{-45}|^2$ cube beamsplitter transmission coefficients are considered equal for a $45^\circ$ and a $-45^\circ$ linear input polarization with respect to the horizontal axis. The reflection coefficients, $|R_{45}|^2$ and $|R_{-45}|^2$, are also considered equal. $p_x^2$ is the polarizer transmission coefficient. Using these relations, the experimental normalized Stokes parameters $S_0^{N(e)}$ and $S_2^{N(e)}$ are retrieved as:

\begin{align}
S_0^{N(e)} & = \frac{I_1}{|T_{45}|^2 T_{\text{tot}}} = \frac{I_1}{T_{\text{tot}}^2} \\
S_2^{N(e)} & = \frac{2 I_2}{p_x^2 |T_{45}|^2} = \frac{I_2}{2 T_{\text{tot}} \sqrt{1 - f}}
\end{align}

Fig. 4. Micrography of the resonator and waveguide. A 1.2 $\mu$m diameter tapered optical fiber is brought near to a toroidal silica microtoroid resonator using a piezoelectric stage.
where $f$ is the power fraction on the $a_y$ axis. The total power, $P_{tot}$, is written as:

$$P_{tot} = a_x^2 + a_y^2 = \frac{P_{1ff}}{|T_{45}|^2} \rightarrow a_x = \sqrt{(1-f)P_{tot}} \text{ and } a_y = \sqrt{fP_{tot}}.$$ 

$P_{1ff}$ is the intensity measurement at D1 off the resonance where the coupling can be neglected ($|T|^2 = 1$).

In Figs. 5(a)-5(c), the normalized $S_0^{N(e)}$ (red curve) and $S_2^{N(e)}$ (green curve) are shown for three coupling conditions (CC). The black dotted lines show the extrema of $S_2^{N(e)}$. The measured parameters, $\Delta S_2^N$, $Q_T$ and their corresponding $Q_0^{(e)}$ and $Q_c^{(e)}$ are presented in Table 1 for the three CC. The $Q_T$ values were obtained by fitting a lorentzian curve to $S_0^{N(e)}$. This can be done
because the FWHM and the center wavelength are the same for both $S_0$ and the transmission $|T|^2$. As it can be seen from the increasing amplitude of $S_2^{N(e)}$ from Fig. 5(a) to Fig. 5(c), the coupling coefficient is also increased, meaning a smaller gap between the waveguide and the cavity.

Using the extracted values of $Q_0^{(e)}$ and $Q_c^{(e)}$ in the proposed model (Eq. (1)-(3)), the calculated $S_0$ (black curve) and $S_2$ (blue curve) are drawn using $f$-parameters equal to 0.4, 0.435 and 0.445 for the first, second and third coupling condition respectively. These changes show the rotation of the polarization between different measurements. The $\phi$ parameter is extracted using the off-resonance value of $S_2^{N(e)}$ where $\cos(\theta + \phi) \to \cos(\phi)$. There is a very good agreement between the experimental and the calculated curves, which shows that the proposed model, despite its simplicity, represents well the polarization changes inside the cavity.

<table>
<thead>
<tr>
<th>CC</th>
<th>$Q_T$</th>
<th>$\Delta S_2^N$</th>
<th>$Q_0^{(e)}$</th>
<th>$Q_c^{(e)}$</th>
<th>$Q_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(2.240 \pm 0.015) \times 10^6$</td>
<td>$0.028 \pm 0.002$</td>
<td>$(2.273 \pm 0.017) \times 10^6$</td>
<td>$(1.58 \pm 0.12) \times 10^8$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(2.198 \pm 0.007) \times 10^6$</td>
<td>$0.115 \pm 0.002$</td>
<td>$(2.333 \pm 0.010) \times 10^6$</td>
<td>$(3.81 \pm 0.08) \times 10^7$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$(1.592 \pm 0.004) \times 10^6$</td>
<td>$0.622 \pm 0.002$</td>
<td>$(2.311 \pm 0.009) \times 10^6$</td>
<td>$(5.11 \pm 0.03) \times 10^6$</td>
<td></td>
</tr>
</tbody>
</table>

It is worth noting the quasi constant value of $Q_0^{(e)}$ for the three coupling conditions. An additional measurement was taken for a very low coupling coefficient where $Q_T \approx Q_0$ gave a $Q_T = (2.26 \pm 0.05) \times 10^6$ which fits well with the presented data. This anticipated behavior shows that only the coupling losses are changed if the coupling conditions change. Thus, the intrinsic losses are only slightly changed by the fiber presence.

### 4. Conclusion

In this paper, we proposed a single scan method to quantitatively extract the intrinsic Q-factor $Q_0$ and the coupling Q-factor $Q_c$ of a WGM or ring-type microcavity, regardless of the coupling regime. This technique is based on a simple Stokes parameters measurement. The theoretical model has been detailed and experimentally verified with a very good agreement. The determination of $Q_0$ and $Q_c$ is accurate within 1 percent if $Q_T$ is higher than $1 \times 10^5$. Compared to the laser sweeping technique [15] which is limited to $Q_T \sim 10^7$, our method can be used for lower $Q_T$ resonators such as integrated microrings. The detection setup requires only simple optical components excluding usually needed fast electronics [13, 14]. This method provides a direct way to determine the relevant quality factors for waveguide-coupled microresonators, and is particularly useful for integrated systems.

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