On the Force Capability of Underactuated Fingers

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Abstract

This paper studies the force capability of a particular class of underactuated fingers. Force capability is defined as the ability to create an external wrench onto a fixed object. The concept of underactuation in robotic fingers, with fewer actuators than degrees of freedom (DOF) through the use of springs and mechanical limits, allows the hand to adjust itself to an irregularly shaped object without complex control strategy and numerous sensors. However, in some configurations, the force distribution in an underactuated finger can degenerate. The finger can no longer apply forces on the object, leading in some cases to the ejection of the latter from the hand. This paper focuses on a 2-DOF finger and studies its ability to seize objects with a secure grasp.

1 Introduction

Since the mid-80s, many robotics laboratories have developed multi-purpose end-effectors, namely robotic hands with the aim of matching the human hand in terms of dexterity and adaptation capabilities. In fact, the idea of copying the human is rather ancient. Hephaestus was said to have created golden mechanical maidservants to assist him. The first modern mechanical grippers, based on a circular translational motion, were developed in the Argonne laboratory to handle nuclear materials with a telemanipulator. Later, many other prototypes were designed but all based either on the latter type of motion or on a simple rotation (clamping motion). The first device that can be referred to as a robotic hand in the modern sense of the term was the end-effector of the Handymen manufactured by General Electric and developed by Ralph Mosher in 1960. In the late 1960s, in Japan and the United States, research on anthropomorphic hands based on designs with three fingers including a so-called “thumb” (a particular finger in opposition) started. However, until now, the human hand remains unmatched despite numerous and interesting attempts. Pioneer designs include: the Utah/MIT hand [1], the Stanford/JPL (Salisbury’s) hand [2], the Belgrade/USC hand [3], the BarrettHand [4], the hands from the DLR [5], or the Okada hand [6]. However, significant efforts have been made to find designs simple enough to be easily built and controlled, in order to obtain practical systems [7], particularly in human prosthetics. Indeed, only one of the hands listed above has been successfully commercialized. To overcome this lack of success — mainly due to the cost of the control architecture needed for complex mechanical systems with often more than ten actuators plus many sensors — a particular emphasis has been placed on the reduction of the number of degrees of freedom, thereby decreasing the number of actuators. In particular, the hands from Nanyang University [8] with one actuator (but seven clutches), the SSL hand [9], the Grasp hand [10], the DIES-DIEM hand [11], the Cassino finger [12], the TBM hand [13], and the KIST gripper [14] (based on a deformable-platform parallel manipulator) have followed this path. On the other hand, very few prototypes involve a smaller number of actuators without decreasing the number of degrees of freedom. This approach, namely underactuation, can be used through the use of passive elements like springs or mechanical limits leading to a mechanical adaptation of the finger to the shape of the object to be grasped [15, 16, 17]. A similar approach consists in using elastic phalanges, which increase the adaptation capability but decrease considerably the strength of the grasp [18].

Underactuation in robotic hands generates intriguing properties. For example, phalanges cannot always apply forces on the grasped object and particular geometrical configurations of the fingers and contact situations can lead to a singular equilibrium as it will be shown in this paper. Thus, a fundamental question should be answered: "when can the finger apply forces on the object grasped?" and how can we increase the occurrence of these situations. We shall illustrate a method to define and determine the force capability of underactuated fingers.
2 2-DOF Underactuated Finger

2.1 Static Equilibrium
A particular design of underactuated finger will be adopted here. This design is a simplified version of the finger that was used in the Mars and Sarah M1 prototypes [19]. Fig. 1 presents the model. The actuation torque $T_a$ is applied to the link a which transmits the effort to the phalanges. A rotational spring $T$ in $O_1$ is used to keep the finger from incoherent motions. The closing process is illustrated in Fig. 2. Notice the mechanical limit that allows a pre-tension of the spring to prevent any undesirable motion of the second phalanx due to its own weight and/or inertial effects, and also prevents hyperflexion of the finger.

![Figure 1: Model of underactuated 2-DOF finger.](image)

In order to determine the configurations where the finger can apply forces to the object grasped, we shall proceed with a quasi-static modeling of the finger. The latter will provide us with the relationship between the input actuator torque and the forces exerted on the object. Equating the input and the output virtual powers, one obtains

$$t^T \omega_a = f^T v$$

where $t$ is the input torque vector exerted by the actuator and the spring, $\omega_a$ is the corresponding velocity vector, $f$ is the vector of contact forces, and $v$ is the velocity of the contact points projected onto the respective normals of the phalanges. Contact forces are assumed to be normal to the phalanges and without friction. More precisely, one has

$$t = \begin{bmatrix} T_a \\ T = -k \Delta \theta_2 \end{bmatrix}, \quad \omega_a = \begin{bmatrix} \dot{\theta}_a \\ \dot{\theta}_2 \end{bmatrix}$$

and

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \quad v = \begin{bmatrix} v_{C_1}^T \ y_1 \\ v_{C_2}^T \ y_2 \end{bmatrix}.$$  

The projected velocities of the contact points can be simply expressed as the product of a Jacobian matrix $J_T$ and the derivatives of the phalanx joint coordinates which is a natural choice, i.e. $v = J_T \dot{\theta}$ or

$$\begin{bmatrix} v_{C_1}^T \ y_1 \\ v_{C_2}^T \ y_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 + l_1 \cos \theta_2 \\ k_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}.$$  

Through differential calculus, one can also relate vector $\omega_a$ to the derivatives of the phalanx joint coordinates defined previously with an actuation Jacobian matrix $J_A$, i.e. $\dot{\theta} = J_A \omega_a$ or

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} \begin{bmatrix} \dot{\theta}_a \\ \dot{\theta}_2 \end{bmatrix}$$

where $X = 1$ and

$$Y = \frac{c[l_1 \sin(\theta_2 - \psi) - a \sin(\theta_1 - \theta_a + \theta_2 - \psi)]}{a[l_1 \sin(\theta_1 - \theta_a) + c \sin(\theta_1 - \theta_a + \theta_2 - \psi)]}.$$  

Finally, one obtains

$$f = J_T^T J_A^T t,$$

which is the equation that provides a practical relationship between the actuator torques and contact forces. The equation is valid if and only if $k_1 k_2 \neq 0$ which is the condition of singularity for the $J_T$ matrix, noting that $J_A$ cannot be singular. The resulting expression for $f$ has been verified with a classical static analysis. If the spring contribution is neglected — which is justified in practice — the analytical expressions are rather simple linear functions of the actuator torque, i.e.

$$f_1 = \frac{(k_2 - h \cos \theta_2)l_1}{ak_1 k_2 (\cot \beta \cos \alpha_1 + \sin \alpha_1)} T_a,$$

$$f_2 = \frac{h}{ak_2 (\cot \beta \cos \alpha_1 + \sin \alpha_1)} T_a$$

where $h = c (\cos(\theta_2 - \psi) - \sin(\theta_2 - \psi) \cot \beta)$ is the distance between point $O_1$ and the intersection of lines.
(OO1) and (P1P2). Also, α1 is the angle between link a and the first phalanx. It can be shown that

$$\cot \beta \cos \alpha_1 + \sin \alpha_1 = \frac{\sin X}{\sin \beta}$$

(10)

where X is the angle between links a and b, i.e.

$$\cos X = \frac{c^2 + l_1^2 - a^2 - b^2 + 2cl_1 \cos(\theta_2 - \psi)}{2ab}.$$ (11)

It is pointed out therefore that if 0 < β < π and 0 < X < π, which corresponds to the general case, denominators of eqs. (8) and (9) are positive. Limit cases in which β = 0 or π will be discussed later.

2.2 Analysis

For the sake of simplicity, the spring will henceforth be neglected. With the results from section 2.1 one can study the condition under which both f1 and f2 are positive, which depends on the geometrical configuration of the finger (described by vector θ) and the contact locations on the phalanges given by k1 and k2. The feasible configurations are presented in Fig. 3 for a set of geometric parameters presented in Table 1. Shaded areas are unstable contact configurations where

$$f_1 < 0 \text{ or } f_2 < 0$$ (12)

Emphasis should be placed on the fact that, with a fully actuated finger, the stable zones will be the whole workspace.

![Figure 3: Positive definite zones for f.](image)

When in an unstable configuration, the closing process will force the finger to lose contact with the proximal phalanx. Contact will only remain with the distal phalanx (Figs. 4 and 5). On Fig. 4, empty arrows indicate mono-directionality of contact forces, objects grasped are fixed in space and can be merged. This phenomenon is particularly clear on the demonstration videos of the LEMUR project gripper [20].

![Figure 4: Stable and unstable configurations with two contact points.](image)

However, an equilibrium position can still be attained but just for a one and unique particular position of contact k2. Static analysis shows that this contact position should be

$$k_2 = e = c \cos \theta_2(\cos(\theta_2 - \psi) - \sin(\theta_2 - \psi) \cot \beta)$$ (13)

with the corresponding contact force being

$$f_2 = \frac{T_a}{a \cos \theta_2 \cos(\theta_1 + \cos \alpha_1 \cot \beta)}.$$ (14)

Note that the term cos θ2 in the denominator of f2 also implies that contact can only be kept for −π/2 < θ2 < π/2 which will be the boundaries of our graphs along with 0 < k2 < l2.

![Figure 5: Stable and unstable one-phalanx contact configurations.](image)

Physically, the above expression implies that the contact force should be located on the projection onto the distal phalanx of intersection of lines (OO1) and (P1P2) (dashed lines on Fig. 5). Indeed, the distal phalanx is subjected to three pure forces, thus equilibrium can only exist if they all intersect in a common point. The existence of this equilibrium point has been noticed in [17], stating that during the sliding of the phalanx along the contact point, the latter tends to converge towards that equilibrium point, which is not however always physically located on the phalanx, i.e. ∃ e | e > l2. Unfortunately, if this behaviour is common, this convergence does not always occur. To demonstrate this point, examine the evolution of the contact location as seen from the phalanx.
It can be easily shown that if only this contact exists and it is fixed in space, one has

$$k_2^2 - k_{2i}^2 + 2d_1 (k_2 \cos \theta_2 - k_{2i} \cos \theta_{2i}) = 0 \quad (15)$$

where $k_{2i}$ and $\theta_{2i}$ are an arbitrary initial configuration, for example, the instant when the contact on the first phalanx is lost. This equation allows us to obtain the precise evolution of the contact position with respect to the evolution of $\theta_2$. The latter itself is determined by the location of the contact with respect to the equilibrium position. If contact is located below the equilibrium point, the finger undergoes an opening motion and thus $\theta_2$ increases. Contact state evolves along the trajectories defined by eq. (15), and then, if the contact trajectory crosses the equilibrium equation defined by eq. (13), the grasp is finally stable, or else contact with the object will be lost, namely one obtains the ejection phenomenon (illustrated in Fig. 6). This phenomenon has been noticed in [17], and is related to the self posture changeability of mechanisms described in [21] but has never been closely studied despite its importance.

![Figure 6: The ejection phenomenon.](image)

Depending on the geometric parameters (Table 1) of the mechanism, one can obtain the final stability of the grasp depending on the initial contact situation, as presented Fig. 7.

| Table 1: Geometric parameters |
|-----------|-----|-----|-----|
| Set | $l_1$ | $l_2$ | $a$ | $b$ | $c$ |
| 1   | 2/3 | $90^\circ$ | 2/3 | 1 | 1/3 |
| 2   | 1   | $4/5$   | $90^\circ$ | 6/5 | 1/3 |

Contact trajectories defined above are indicated by dotted lines on Figs. 7 and 8, the arrows indicate the direction of the contact evolution, and dashed lines indicate the equilibrium equation. The contact state — defined by the pair $(\theta_2, k_2)$ — slides along a dotted line until either crossing the equilibrium curve (resulting in a stable grasp) or leaving the boundaries of the graph (contact lost, ejection). Given the evolution of $\theta_2$, one can separate the equilibrium curve in two distinct stable parts, one attracting limit (the lower part on the graph) and a repulsive limit (upper part), the transition between the two modes being the tangent point between the contact trajectories and equilibrium curve, which is often close to the maximum of the equilibrium equation.

![Figure 7: Final stability of the grasp with one phalanx contact (parameter set 1).](image)

However, when the equilibrium point leaves physically the phalanx ($e > l_2$), another unstable frontier appears, see for example Fig. 8. Contact situations under the equilibrium curve and on the right hand side of the stability limit (shaded area number 2) are unstable because the contact trajectories cross the boundary $k_2/l_2 = 1$ before attaining the attractive limit. The latter is not physically on the phalanx for this trajectory and this parameter set. This corresponds to the ejection phenomenon depicted on Fig. 6, ejection for a closing motion ($\theta_2$ decreases) of the distal phalanx. Ejection for an opening motion ($\theta_2$ increases) of the distal phalanx can also occur (shaded area number (1) on Fig.8), in fact on Fig. 7 it is the only form of ejection possible. If the distal phalanx tends to open, the motion of link $a$ (thus motion as seen from the actuator) is still a closing process. Nevertheless, contact situation will open the finger. This can and should be prevented.
with mechanical limits. For example, a mechanical limit such as \( \theta_2 > 0 \) will eliminate the ejection of type 1 since the point of the contact trajectories with vertical tangent are defined by

\[
\begin{align*}
\theta_2 &= 0 \\
\frac{k_2}{k_2} - \frac{k_2}{k_2} + 2d_1 (k_2 - k_2 \cos \theta_2) &= 0
\end{align*}
\]

(16)

Furthermore, another limit such as \( \theta_2 < \pi/2 \) should also be used since contact forces can only be kept for \(-\pi/2 < \theta_2 < \pi/2\) as discussed after eq. 14. The large regions of instability in Figs. 7 and 8 should be moderated, because one should remember that unstable regions on these figures correspond to stable two-phalax contact grasps. Each stability region is dual to the other. Hence, in most cases, i.e. when the contact is initially established with the proximal phalax, the final grasp will be stable with the notable exception of the second type of unstable region discovered in Fig. 8. This type of instability should be therefore avoided as much as possible through suitable design. Moreover, if joint limits are used, they act as stable limits (however not attractive).

In conclusion, the equation of the equilibrium point is of utmost importance for the stability of the grasp and allows us to choose between two-phalax (power) or one-phalax (pinch) grasps. One shall then proceed with the detailed study of this function.

### 2.3 Equilibrium Point Equation

The equilibrium equation (13) can be written as

\[
e = c \cos \theta_2 (\cos (\theta_2 - \psi) - K \sin (\theta_2 - \psi))
\]

(17)

where \( K = \cot \beta \) describes the opening of the four-bar linkage (\( OOh_iP_iP_2 \)) and can also be expressed as

\[
K = \frac{cS_{\theta_2 - \psi} \sqrt{4a^2b^2 - N^2} + M(l_1 + cC_{\theta_2 - \psi})}{-(l_1 + cC_{\theta_2 - \psi}) \sqrt{4a^2b^2 - N^2} + Ml_1S_{\theta_2 - \psi}}
\]

(18)

where \( S_\theta \) and \( C_\theta \) stand respectively for \( \sin \theta \) and \( \cos \theta \), and with

- \( M = -l_1(l_1 + 2c \cos (\theta_2 - \psi)) + a^2 - b^2 - c^2 \)
- \( N = l_1(l_1 + 2c \cos (\theta_2 - \psi)) - a^2 - b^2 + c^2 \)

The equilibrium location equation (17) is primarily function of two parameters, namely the ratio \( c/a \) and the angle \( \psi \). Their respective influence is shown in Fig. 9. If the spring is not neglected, deviation is observed depending on the ratio between the input torque and the spring torque.

The limit ratio \( c/a = 1 \) (i.e. \( \beta = 0 \) in all configurations) should be avoided, since, in that case, the one-phalax contact case is always unstable, due to the impossible equilibrium of the distal phalax. The latter is subject to three forces, two of them being always parallel. The equilibrium point is thus pushed to infinity. In consequence, a simple 4-bar parallelogram linkage should not be used (Fig. 10(a)). Even in the case of two-phalax contact, this design is inappropriate because components of the contact force vector \( f \) are

\[
f_1 = -\frac{b \cos \theta_2}{k_1} T_a \quad \text{and} \quad f_2 = \frac{T_a}{k_2}
\]

(19)

Hence, \( f_1 \) is almost always negative (except in extreme configurations where \( \theta_2 > \pi/2 \)). Contact with the proximal phalax cannot usually be maintained resulting in a distal-phalax only contact which has an impossible equilibrium point. Therefore, the finger ejects the object. Moreover, if \( c/a > 1 \) the closing process will tend to align points \( O, P_1, \) and \( P_2, \) resulting in a configuration where \( f = 0 \) so contact cannot be sustained.

On the other hand, one can use the previous results to design a finger that minimizes the ejection phenomenon through numerical analysis, resulting in an optimally stable design presented in table 2 (parameters have been approximated to the closest simple ratio) and Fig. 10(b). The stability limits are presented in Fig. 11. If \( 0^\circ < \theta_2 < 90^\circ \) joint limits are used, the finger can always provide a stable grasp.
Figure 10: Almost always unstable (a) and optimal (b) design of underactuated finger.

with an approximately equal probability of one- and two-phalanx contact.

<table>
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<th>Set</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$\psi$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
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<td>3</td>
<td>1</td>
<td>$3/4$</td>
<td>$54^\circ$</td>
<td>$3/4$</td>
<td>1</td>
<td>$1/3$</td>
</tr>
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</table>

Figure 11: Final stability of the grasp with one phalanx contact (optimal parameters set).

Figure 12: CAD model of an underactuated gripper.

3 Conclusions

This paper has detailed and analyzed in detail the force capability of a particular underactuated finger. Underactuation considerably simplifies the control process of robotics hands, it also brings a new kind of unstable grasps that must be studied and prevented. Other work not included here because of lack of space includes cylindrical and plane contacts, taking into account of friction, grasp force distribution, and other actuation mechanisms. Ongoing research is focusing on three-phalanx fingers and underactuated finger synthesis using the presented results.

Acknowledgments

The authors wish to thank Mathieu Goulet and Pierre-Luc Richard for designing the plastic models in Pro/ENGINEER. The financial support of NSERC and the Canada Research Chair Program is acknowledged.

References


