Optimal Design of 2-Phalanx Underactuated Fingers

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Abstract

This paper presents the optimal design of underactuated fingers considering two issues: the force isotropy of the grasp, namely its ability to generate a uniform pressure on the object seized, and its stability. First, the force isotropy property is defined and a method to achieve the latter is presented. Its robustness with respect to undesired variations of the design parameters is also discussed. Second, guidelines to minimize or completely avoid ejection are presented as well as their influence on the desired optimality. In this work, a stable grasp means that the finger is in static equilibrium. The occurrence of the situations where one phalanx force is negative should be decreased (prevention may be impossible [1]) and the ejection avoided. The aim of this paper is to elaborate simple and practical design rules to obtain stable optimal fingers. Previous analyses [2, 3] have described the issue of ejection for underactuated fingers, namely that a grasp sequence can degenerate into ejecting the object, and a numerical minimization was used to obtain effective finger designs. In this paper, the concept of isotropic grasp as an optimality criterion, and also, a method to either minimize or completely eliminate ejection if a limited range of motion is acceptable, are presented. Emphasis should be placed on the fact that the optimal ejection-free design method presented in this paper differs from usual optimization, since one does not maximize a criterion but matches it exactly. The designs resulting from the method proposed are force isotropic and ejection free. Results are given for both tendon-pulley and linkage transmission.

1 Introduction

Until now, the human hand remains unmatched despite numerous and interesting attempts. Pioneer designs include: the Utah/MIT hand [4], the Stanford/JPL (Salisbury’s) hand [5], the Belgrade/USC hand [6], the BarrettHand [7], the hands from the DLR [8], and many others. However, significant efforts have been made to find designs that are simple enough to be easily built and controlled, in order to obtain practical systems [9], particularly in human prosthetics. Indeed, only one of the hands listed above has been successfully commercialized. The lack of success of these complex devices is mainly due to the cost of the control architecture with often more than ten actuators plus many sensors. In order to overcome these limitations, a particular emphasis has been placed on the reduction of the number of degrees of freedom, thereby decreasing the number of actuators. In particular, the DIES-DIEM hand [10], the Cassino finger [11] have followed this path. On the other hand, very few prototypes involve a smaller number of actuators without decreasing the number of degrees of freedom. This approach, referred to as underactuation can be implemented through the use of passive elements like springs or mechanical limits leading to a mechanical adaptation of the finger to the shape of the object to be grasped [12, 13, 14]. The idea to approach the spatial complement of the shape of an object to ensure a distributed and adaptive grasp is rather common in biologically-inspired robotics: e.g. snake robots or elephant trunks. These systems belong to what has been defined as the Frenet-Serret manipulators intended for whole-arm manipulation. The present analysis is based on [1] where the grasp stability of 2-phalanx underactuated finger was studied. In this paper, we propose to apply these results to the design of optimal fingers.

2 Underactuation

Underactuation in robotic fingers is different from the concept of underactuation usually presented in robotic systems and both notions should not be confused. Underactuated fingers generally use elastic elements in the design of their “unactuated” joints. Thus, one should rather think of these joints as uncontrollable or passively driven instead of unactuated. In most underactuated fingers, the actuation wrench $T_a$ is applied to the input of the finger and is transmitted to the phalanges through suitable mechanical elements, e.g. four-bar linkages, pulleys and tendons, gears, etc. Passive elements are used to kinematically constrain the finger and ensure the shape-adaptation of the finger to the object grasped. An example of underactuated 2-phalanx finger using a linkage and its closing sequence are illustrated in Fig. 1. The
actuation torque $T_a$ is applied to the first link which transmits the effort to both phalanges. Notice the mechanical limit that allows a pre-loading of the spring to prevent any undesirable motion of the second phalanx due to its own weight and/or inertial effects, and also to prevent hyperextension of the finger. Springs are useful for keeping the finger from incoherent motion, but when the grasp sequence is complete, they still oppose the actuator force.

Two considerations will form the guidelines of the paper: first, the grasp has to be stable in the sense that ejection should be prevented, second, differences between the phalanx forces should be kept to a minimum. It has been proven [1] that no underactuated finger using classical linkage or tendon transmission (both models are presented in Fig. 2) can always apply forces on a fixed object with both phalanges for every possible configuration. An ideal grasping sequence as illustrated in Fig. 1 does not always occur for in the final configuration, some phalanx forces may be negative. If the proximal phalanx force is negative the latter phalanx will lose contact with the object. Then, another step in the grasping sequence will take place: the distal phalanx corresponding to a positive force will slide on the object surface. This sliding process will continue until either a stable configuration is achieved (with a zero proximal phalanx force), or joint limits are met (stable situation, but the shape adaptation is less effective), or the last phalanx will curl away and loose contact with the object (ejection, illustrated in Fig. 3). If the distal phalanx force is negative (impossible in the case of tendon-actuated fingers), the finger’s configuration corresponds to hyperflexion/hyperextension and the contact is lost with no stable grasp possible. However, these situations are usually made impossible by design and joint limits.

![Figure 1: Closing sequence of a 2-phalanx finger with linkage transmission.](image1)

**Figure 1:** Closing sequence of a 2-phalanx finger with linkage transmission.

### 3 Force Properties and Ejection

Force isotropy is defined as the property of a finger to generate equal contact forces on all its phalanges. This property is of the utmost importance to prevent damages to the grasped object resulting from an unbalanced grasping force distribution. Since the ratio between the phalanx forces is independent from the control variables, this issue must be addressed at the design stage. For instance, the work presented in [12], despite its importance (one of the first attempts with [15] to formalize underactuation for grasping), did not point out that the proposed optimal design is only force isotropic for one and only one configuration, i.e. when

$$\begin{cases}
\theta_i = 0 & i > 1 \\
and \quad k_i = l_i/2 & i > 0
\end{cases} \quad (1)$$

However, this shortcoming was known and verbally acknowledged by one of the authors of [12]. In fact, despite having correctly established, in a very elegant manner, the geometric condition for such a property (quadratic variation of the pulley radii), this optimal design even lacks the isotropic property because of integer approximation as has been established by the authors using the results of [16]. It can be shown that this design generates an equal force but only one every two phalanges. This illustrates how a slight difference (presented in Fig. 4) in the design parameters can have a tremendous impact in terms of the objectives. As a numerical indication, the ratio between the area under the curves in Fig. 4 and the area between them is more than 20 to 1. This shows the need not only to focus on isotropy itself but also on the robustness of this concept, since an exact design is anyway impossible, the optimal design objective should also be to obtain a robust design in the sense of getting geometrical constraints that are as robust to errors as possible. Force design [17] or kinematic design [18] may be used for underactuated fingers, but the first one seems more promising. The later only tries to optimize the finger...
workspace without any knowledge of force properties. Indices to characterize the finger properties inside the workspace have been proposed [19], similarly to serial robot kinematics, but they are not dedicated to finger design and thus may be inappropriate. Kinematic properties like the dexterity are almost irrelevant from a grasping point of view, where forces are the main focus. The approach in [17] is much more relevant and is based on the assumption that the finger motion itself is far less important than the finger force properties during the grasp. This paper acknowledges this statement and follows the same philosophy. Other considerations may also have an important impact on the quality of the grasp, namely one can use limits on the phalanx forces so that the latter are neither too small nor too large, despite being equal (isotropy) for example. The Direct Optimal Problem (DOP) and the Inverse Optimal Problem (IOP) should also be precisely defined. The first problem is to define what state of a system is actually optimal with respect to a certain criterion. The latter consists in designing the system so that a particular task will be optimally performed. For robotic fingers, one aim is to have a force isotropic configuration corresponding to a typically grasped object. Furthermore the final stability of the grasp must be ensured. Combining stability, force-isotropy and acceptable force range may not be simple. The analytical expression of the normal contact forces for a general 2-phalanx underactuated finger has been established in [2], namely

$$ f = \left[ \frac{k_2(1+R)+R l_1 \cos \theta_2}{k_1 k_2} T_a - \frac{k_1 l_1}{k_1 k_2} \cos \theta_2 T_2 \right] $$  \hspace{1cm} \text{(2)}

where $T_2 = K \theta_2 T_a$ is the spring torque, note that the spring stiffness has been related to the actuation torque $T_a$ and a zero neutral position of the spring has been chosen. Additionally,

$$ R = \begin{cases} r_2/r_1 & \text{with a tendon transmission} \\ r_2/(h + l_1) & \text{with a linkage transmission} \end{cases} $$  \hspace{1cm} \text{(3)}

where $h$ is the directed distance between point $O_1$ and the geometric intersection of lines $(O_1 O)$ and $(P_1 P_2)$, i.e.

$$ h = c (\cos(\theta_2 - \psi) - \sin(\theta_2 - \psi) \cot \beta). $$  \hspace{1cm} \text{(4)}

In the latter equation, $\cot \beta$ can be expressed analytically as established in [16].

4 Optimal Design

4.1 Force Isotropic Design

To obtain a force isotropic design, one has to guarantee that both phalanx forces are equal. Obviously, such a characteristic is only local, i.e., it depends on the finger configuration defined by the angle $\theta_2$ and the contact locations $k_1$ and $k_2$. Therefore, if the object is moved in the Cartesian space (with the exception of a rotation around point $O$), this property no longer holds true. For tendon-actuated fingers, the equation $f_1 = f_2$ reduces to

$$ r_1 (k_2 - GK \theta_2) = r_2 G $$  \hspace{1cm} \text{(5)}

where $G = k_1 + l_1 \cos \theta_2 + k_2$. With $\alpha = r_2/r_1$ one obtains

$$ \alpha = -K \theta_2 + \frac{k_2}{k_1 + l_1 \cos \theta_2 + k_1} $$  \hspace{1cm} \text{(6)}

This pulley radius ratio corresponds to a force isotropic contact situation $(k_1, k_2, \theta_2)$. For mechanically actuated finger, the force isotropy equation is

$$ (h + l_1) (k_2 - GK \theta_2) = h G $$  \hspace{1cm} \text{(7)}

Hence, very similarly, the optimal transmission coefficient in this case is

$$ \frac{h}{h + l_1} = -K \theta_2 + \frac{k_2}{k_1 + l_1 \cos \theta_2 + k_1} $$  \hspace{1cm} \text{(8)}

However in this case, another step is necessary since $h$ is a function of the design parameters and the angle $\theta_2$. Furthermore, many design variables are available to satisfy the latter equation, namely $a$, $b$, $c$, and $\psi$. Similarly, in the case of the tendon-actuated finger, the ratio of the pulley radii was specified, not each radius independently. Therefore, some additional constraints of various nature can be satisfied: an upper bound on the dimensions, standard link lengths, commercially available pulleys, or machining simplicity. For instance, with a mechanically actuated finger, if one chooses $a = b$, a known $c$ (e.g. resulting from minimal distance considerations) and $\psi = \pi/2$, $a$ is completely defined as

$$ a = \frac{2l_1 \sin \theta_2 + c^2 + l_1^2 \sqrt{A}}{2B} $$  \hspace{1cm} \text{(9)}
with

\[ A = (C - 1)((C - 1)e^2 + 2cC_l \sin \theta_2) + C^2l_1^2 \]  
\[ B = -e^2 + Ce^2 - l_1 c \sin \theta_2 + 2C_l c \sin \theta_2 + Cl_1^2 \]  
\[ C = -K \theta_2 + \frac{k_2}{k_2 + l_1 \cos \theta_2 + k_1} \]

As can be seen the analytical force isotropy condition with linkages instead of pulley-tendons, is much more complicated. However, this condition can be obtained explicitly (as opposed to numerically) and the method in both cases is identical. These expressions allow to solve the IOP, which is particularly useful when designing underactuated fingers. The solution of the DOP is simply defined by eqs. 5 and 7 respectively, for each kind of transmission. The force isotropic configurations are defined by these equations for a particular set of design parameters, and define a 2D surface in the contact space \((k_1, k_2, \theta_2)\), which will be referred to as the isotropic surface. An example is illustrated in Fig. 5.

![Force Isotropic Surface](image)

**Figure 5:** Force isotropic locus for a tendon-actuated finger defined by \(l_2/l_1 = 2/3, r_2/r_1 = 1/3\), and \(K = 0\).

This isotropic surface is the solution to the DOP, and is also related to the IOP which aims at finding the design parameters defining such a surface passing through a predefined point. It should be however noted that this surface is very sensitive to design parameter variations, one has

\[ \frac{\partial(f_1/f_2)}{\partial \alpha} = -\frac{k_2}{k_1(\alpha + K \theta_2)^2} \]

(13)

for tendon-actuated mechanism, the same result holds for linkage transmission by replacing \(\alpha\) with \(h/(h+l_1)\). Therefore, since \(k_1\) and \(k_2\) are usually of the same order of magnitude, and since both \(\alpha\) and \(K\) are usually small, a slight change in the transmission factor implies a significant change of the isotropic surface. For instance, if the same parameters that were used to plot Fig. 5 are used, the volume between the two surfaces defined by \(f_1 = 0.9f_2\) and \(f_1 = 1.1f_2\), i.e. a 10% tolerance on isotropy, is about 0.3% of the total volume of the plotting space. Indeed, the contact forces present high variations inside the finger workspace. The statistical characteristics of

\[ \kappa = \sqrt{\left(\frac{f_2 - f_1}{f_2}\right)^2} \]

(14)

can illustrate this point. For instance, using the finger defined in Fig. 5, and even for limited variations on the contact situation, i.e.

\[ \left\{ \begin{array}{l} 0 < \theta_2 < \pi/2 \\
 l_i/4 < k_i < 3l_i/4 \quad i = 1, 2 \end{array} \right. \]

(15)

one obtains

\[ \begin{aligned}
 & \text{max } \kappa \approx 2.99 \\
 & \text{min } \kappa = 0 \quad \text{(isotropic situations)} \\
 & \pi = .47
\end{aligned} \]

(16)

Hence, even with this limited workspace, a mean variation of 47% is to be expected. Therefore, the isotropic property is not very robust with respect to design parameters tolerance. Note that the spring can help to reduce this sensitivity by decreasing the factor established in eq. (13). However, the spring stiffness opposes the actuation torque and thus, the grasp forces, and a compromise must be found between isotropy and contact force magnitudes (usually associated with a secure grasp). In [12], the authors neglected the influence of the springs (a return cable in their case) when solving the IOP and this explains the highly sensitive design they obtain, as it has been illustrated in Fig. 4. In this section, the force isotropic design of an underactuated finger with either tendon or linkage transmissions has been presented and studied. It should however be noted that the results given are valid only for a single point of contact on both phalanges (which can be the edges of a polygon or points on a cylinder).

### 4.2 Guidelines to prevent ejection

Another important property of underactuated fingers that should be analyzed is their robustness against ejection. Indeed, if one obtains an isotropic and therefore stable design for a particular contact set \((k_1, k_2, \theta_2)\), it may be of interest that the finger is also robust with respect to ejection around this isotropic point, in order to ensure that a deviation from this configuration does not lead to an unstable grasp. The final aim is to guaranty stability for all grasps if possible and satisfy certain “quality” based indices like the isotropy. An index that can be used to ensure the grasp stability, even if the proximal contact is lost, is:

\[ \mu^* = \frac{\int_W \delta^*(k_2, \theta_2)dk_2d\theta_2}{\int_W dk_2d\theta_2} \]

(17)

where \(\delta^*(k_2, \theta_2)\) is a Kronecker-like symbol for characterizing the stability of the contact situation:

\[ \delta^*(k_2, \theta_2) = \left\{ \begin{array}{l} 1 \quad \text{if the final grasp is stable} \\
 0 \quad \text{otherwise} \end{array} \right. \]

(18)
This index is the ratio between the stable (clear) and unstable (grey shaded) areas in the grasp-state plane of the finger, illustrated in Fig. 6 for the same parameter set than Fig. 5 and considering a single point of contact. Other indices can characterize the ability of the finger to generate full-phalanx grasps, i.e. grasps with all phalanx forces positive, as introduced in [16]. An introduction of the grasp-state plane has been presented in [2], the reader is referred to the latter reference for more information. As a brief reminder, once contact with the proximal phalanx is lost, the sliding motion undergone by the distal phalanx is described by the evolution of the contact situation \((k_2, \theta_2)\) on a contact trajectory, identified in Fig. 6 by dotted curves. Each curve corresponds to a particular point of contact in the Cartesian space. This sliding motion is finished either by attaining a stable situation (blue curve labeled “equilibrium curve” in Fig. 6) or by ejecting the object \((k_2 > l_2)\). The evolution of the contact situation on its corresponding contact trajectory is defined with respect to the equilibrium curve. Two different kinds of ejection have been identified, namely opening- and closing-ejection. Contour plots of the index \(\mu^*\) are illustrated in Figs. 7 and 8 for respectively, a tendon-actuated and mechanically actuated finger. As it can be seen on the figures, both plots are very similar. General guidelines arise from these figures: for a given performance value and phalanx lengths, there are typically two ratios \(r_2/r_1\) satisfying the latter: the highest ratio presents closing-ejection in the workspace while the smallest has opening-ejection. These figures can also be used to find the optimal pulley radius ratio for given phalanx lengths. Please note that in these examples, the full \([-\pi, \pi]\) range of motion of angle \(\theta_2\) has been considered. If this range is not physically possible the latter should be accordingly reduced and effects of the mechanical limits should be taken into account.

For instance a 2-phalanx version of the Soft Gripper [12] achieves a score of 0.28 while with the same phalanx lengths a pulley ratio of 0.55 would increase the latter to 0.45, a 60% gain. However, the authors of [12] tried to obtain a force isotropic design and did not consider ejection. This fact explains such a difference. Furthermore, if mechanical limits restrain the achievable range of angle \(\theta_2\) between 0 and \(\pi/2\), both designs obtain a perfect 1 score, illustrating that ejection can be eliminated in certain cases.

To prevent opening-ejection, one has to set mechanical limits such as \(0 < \theta_2\). To prevent closing-ejection, the finger must be designed so that either the equilibrium curve (Fig. 6) is always physically on the phalanx, or that both solutions of \(e(\theta_2) = l_2\) (\(e\) denotes the mathematical expression of the equilibrium curve), if they exist, correspond to \(\theta_2 < 0\). For tendon actuated
finger, since the solution of the equation \(e(\theta_2) = l_2\) is
\[
\theta_2 = \pm \arccos \left( \frac{\sqrt{l_2^2 - l_1^2}}{l_2} \right), \tag{19}
\]
the only solution to avoid closing-ejection is to ensure that \(\max_l [\theta_2] < l_2\). Furthermore, another mechanical limit such as \(\theta_2 < \pi/2\) should be set for the contact to be made with the front of the finger. Since mechanical limits act like additional equilibrium curves (described by \(\theta_2 = \text{cst}\) in the grasp-state plane), contact situations that would normally intersect them will result in stable grasps. Therefore, one can design a finger that eliminates ejection for a single point of contact. Other contact hypotheses could also be of interest and are not covered here, namely different local geometry of the contact (circular, linear, etc.), friction, etc. If the aforementioned conditions cannot be met to prevent ejection, a minimization of this phenomenon, measured with the index \(\mu^\star\) through numerical or algebraic analysis should be performed. There is of course a relationship between the condition on the equilibrium curve to eliminate closing-ejection and the optimal transmission ratio resulting from the previous section. It is possible that this optimal ratio corresponds to a finger presenting closing-ejection in its workspace. To avoid this situation, the design of the finger should be decomposed in two consecutive steps:

1. choose \(\alpha\) according to the force isotropic optimal criterion, this implies the choice of \(l_1\)
2. choose \(l_2\) in order for the equilibrium curve to stay on the phalanx, i.e. \(l_2 > \alpha l_1/(1 - \alpha)\)

A similar technique can be used for mechanical fingers, but again, the expressions are much more complicated.

5 Conclusions

In this paper, the optimal design of 2-phalanx underactuated fingers has been presented. The results obtained can be used to design fingers using either linkages or pulleys as transmission mechanisms. Two issues have been considered as an optimality criterion: the isotropy of the grasp (i.e. both phalanx forces are identical) and the overall grasp stability of the finger with respect to ejection. The aim of this paper is to provide practical tools for the design of underactuated finger. Previously, this design was mainly driven by the designer’s inspiration which can result in very good designs as well as very poor results. If the rules presented in this paper are followed, one can ensure the optimality of the design with respect to the grasping characteristics.

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